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TITULO: Capitulo 8

THE RATE OF INTEREST AND THE OPTIMUM PROPENSITY TO CONSUME

SACADO DEL LIBRO: READINGS IN BUSINESS CYCLE THEORY

C E N D E S

CURSO: TEORIA ECONOMICA

PROF.: JUAN P. PEREZ CASTILLO

(solo para distribución interna)

Caracas, febrero de 1966

THE RATE OF INTEREST AND THE OPTIMUM PROPENSITY TO CONSUME*

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1. By introducing liquidity preference into the theory of interest Mr. Keynes has provided us with an analytical apparatus of great power to attack problems which hitherto have successfully resisted the intrusion of the economic theorist. In this paper I propose first to elucidate the way in which liquidity preference co-operates with the marginal efficiency of investment and with the propensity to consume in determining the rate of interest and to point out how both the traditional and Mr. Keynes's theory are but special cases of a more general theory. Further I propose to show how the analytical apparatus created by Mr. Keynes can be used to handle the problem which bothered the under-consumption theorists since the time of Malthus and Sismondi.

The economic relations by which the rate of interest is determined can be represented by a system of four equations.[1]

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*Economica, Volume V (New Series), Number 17, February 1938, pages 12-32. Reprinted by the courtesy of The London School of Economics and Political Science and the author.

‖University of Chicago.

[1]A similar system of equations has been given for the first time by Reddaway, "The General Theory of Employment, Interest and Money," The Economic Record, June, 1936, p.35. While writing this there has come to my notice a forthcoming paper of Dr. Hicks on "Mr. Keynes and the Classics," in the meanwhile published in Econometrica, April, 1937, which treats the subject in a similar and very elegant way. The form chosen in my paper seems, however, more adapted for the study of the problems it is concerned with. Cf. also Harrod, "Mr. Keynes and Traditional Theory," Econometrica, January, 1937.

The first of these equations is the function relating the amount of money held in cash balances to the rate of interest and to income. This is the liquidity preference function. If M is the amount of money held by the individuals, Y their total income and i the rate of interest we have:[1]

$$M = L(i, Y) \qquad (1)$$

It is convenient to take M and Y as measured in terms of wage-units, or of any other numéraire. Thus Y is the real income while M is the real value of the cash balances, both in terms of the numéraire chosen. This presupposes, of course, that the ratio of the price of each commodity or service to the price of the commodity or service which is chosen as the numéraire is given. These ratios may be thought of as determined by the Walrasian or Paretian system of equations of general economic equilibrium. Thus index numbers are not involved in this procedure. We assume that the real value, as defined, of cash balances decreases (or, in the limiting case, remains constant) in response to an increase of the rate of interest and that it increases (or, in the limiting case, remains constant) in response to an increase of real income, i.e., $L_i \leq 0$ and $L_y \geq 0$.

The second equation expresses the propensity to consume. The total expenditure on consumption depends on the total income and, possibly, on the rate of interest. Denoting by C the total expenditure on consumption during a unit of time, we have the function:[2]

$$C = \phi\,(Y, i) \qquad (2)$$

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[1] This function is obtained by summation of the liquidity preference functions of the individuals in the same way as a market demand function is obtained from the demand functions of the individuals. It holds only for a given distribution of incomes.

[2] This function is the sum of the functions expressing the propensity to consume for each individual. It holds only for a given distribution of incomes

where C and Y are measured in wage-units (or in some other numéraire chosen). The expenditure on consumption increases in response to an increase of income, though less than the income, i.e., $0<\phi_y<1$, while no general rule can be stated as to the reaction of this expenditure to a change in the rate of interest, so that $\phi_i \gtreqless 0$.

The investment function which relates the amount invested per unit of time to the rate of interest and to the expenditure on consumption provides us with a third equation. If I is the investment per unit of time the function is:

$$I = F(i, C) \tag{3}$$

Both I and C are measured in wage-units. The investment function is based on the theorem that the amount of investment per unit of time is such as to equalise the rate of net return on that investment (the marginal efficiency in Mr. Keynes' terminology) to the rate of interest. This rate of net return is derived from the rate of net return (marginal efficiency) on capital but it is not identical with it.[1] The lower the rate of interest the larger the investment per unit of time, i.e., $F_i<0$. Investment per unit of time depends, however, not only on the rate of interest but also on the expenditure on consumption. For the demand for investment goods is derived from the demand for consumers' goods. The smaller the expenditure on consumption the smaller is the demand for consumers' goods and, consequently, the lower is the rate of net return on investment. Thus, the rate of interest being constant, investment per unit of time is the larger the larger the total expenditure on consumption, i.e., $F_c>0$.

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[1] They are frequently confused. However, the marginal efficiency of capital relates the rate of net return to a stock of capital while the marginal efficiency of investment relates it to a stream of investment per unit of time. As to how the marginal efficiency of investment is related to the marginal efficiency of capital cf. a forthcoming paper by Mr. Lerner. It also ought to be observed that the investment function holds only for a given capital equipment and for a given distribution of the expenditure for consumption between the different industries.

Finally we have the identity:

$$Y = C + I$$

which provides us with the fourth equation.[1]

If the amount of money M (in wage-units) is given these four equations determine the four unknowns, i, C, I and Y. Alternatively, i may be regarded as given (for instance, fixed by the banking system) and M as determined by our system of equations. These equations determine also the income-velocity of circulation

of money which is $\frac{Y}{M}$.[2] It must, however, be remembered that C, I and Y

are measured in terms of a numéraire (wage-units). If we want them to be expressed in money we need an additional equation which expresses the money price of the commodity or service chosen as numéraire (a unit of labour in our case). If W is this money price and Q the quantity of money we have:

$$Q = wM$$

which is equivalent to the traditional equation of the quantity theory of money.

3. The process of determination of the rate of interest according to the four equations above is illustrated by the three following diagrams.

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[1]This identity is the sum of the budget equations of the individuals. It can also be written in the form Y - C = I which expresses the equality of investment and the excess of income over expenditure on consumption, i.e., saving. The identical equality of investment and saving holds for investment and saving actually performed. Investment or saving decisions can be different. The identity above states, however, that, whatever the decisions, income is bound to change so as to make equal saving and investment actually realised.

[2]It is interesting to notice that the income-velocity resulting

Fig. 1 represents the relation between the demand for cash balances and the rate of interest. The quantity of money (in wage-units) being measured along the axis 0M and the rate of interest along the axis 0i, we have a family of liquidity preference curves: one for each level of total income (measured in wage units).

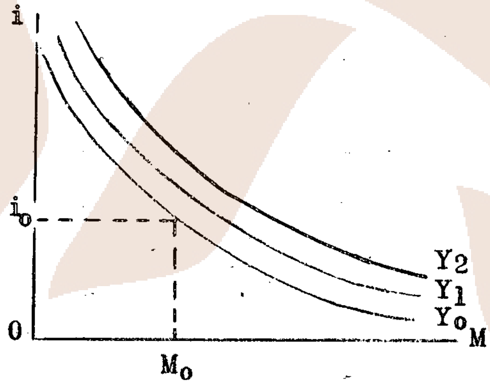

Fig. 1

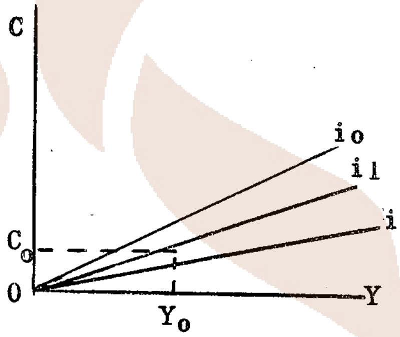

Fig. 2

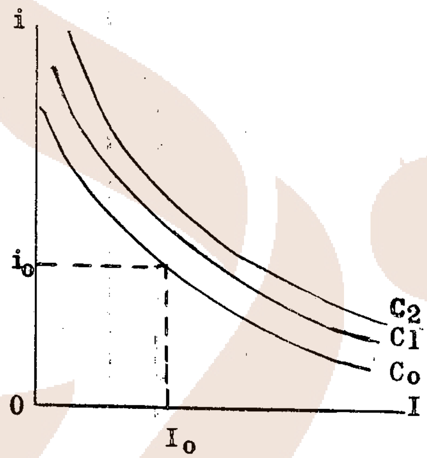

Fig. 3

The greater the total income the higher up is the position of the corresponding curve.

Cont. footnote Nº 2. From these equations is the "hybrid" corresponding to the definition of Professor Pigou (cf. Industrial Fluctuations, 1927, p. 152) and of Mr. Robertson (Money, new edition, 1932, p. 38) and not the ratio of income to income deposits only which Mr. Keynes calls income-velocity (cf. A Treatise on Money, Vol. II, pp. 24-25).

Further we have a family of curves (one for each rate of interest) representing the relation between income and expenditure on consumption (Fig.2). Income is measured along OY and expenditure on consumption along OC.

The relation between investment and the rate of interest is represented by Fig.3. Measuring investment per unit of time along the axis OI and the rate of interest along Oi we have a family of curves indicating the investment corresponding to each value of the rate of interest.

These curves represent the marginal net return (marginal efficiency) of each amount of investment per unit of time. It is important to notice that there is a separate curve for each level of expenditure on consumption. The greater the expenditure on consumption the higher up is the position of the corresponding curve.

To study the process of determination of the rate of interest let us start with a given amount of money (OM_0 in Fig.1) which is kept constant throughout the process and with a given initial income Y_0. The position of the liquidity preference curve being determined by the level of income (in Fig.1 the curve corresponding to the income Y_0), the amount of money determines the rate of interest, say i_0. This rate of interest determines the position of the curve in Fig.2 representing the propensity to consume. This position being determined, we get the expenditure on consumption C_0 corresponding to the initial income Y_0. The expenditure on consumption being given, the position of the marginal efficiency curve in Fig. 3 is determined (i.e., the curve corresponding to C_0). When this position is determined the rate of interest i_0 determines the amount I_0 of investment per unit of time. We have thus the expenditure on consumption and the amount of investment. But the sum of these two is equal to the total income (vide equation 4). If it happens so that $C_0 + I_0$ is equal to the initial income Y_0 the system is in equilibrium. Otherwise the liquidity preference curve in Fig.1 changes its position so as to correspond to the new level of income $C_0 + I_0$. This gives us a new rate of interest. As a result of this and of the changed income we get a new level of expenditure on consumption. This in turn changes the position of the marginal efficiency curve in Fig. 3 and the new rate of interest determines another amount of investment which, together with the expenditure on consumption, determines a third level of total income. As a result the liquidity preference curve shifts again, etc. This process of mutual adjustment goes on until the curves in our three diagrams have reached a position compatible with each other and with the quantity of money given, i.e., until equilibrium is attained.[1]

4. Let us now consider how changes in the curves of the marginal efficiency of investment and in the curves representing the propensity to consume affect the rate of interest.

If the marginal efficiency curves are all shifted upwards (which, ultimately, must be due to an increase of the marginal net productivity of capital), then a larger amount of investment corresponds to any given rate of interest and expenditure on consumption. Therefore total income increases and the curve of liquidity preference in Fig. 1 shifts upwards. This causes a rise of the rate of interest. Thus, just as in the traditional theory, an increase in the marginal productivity of capital is accompanied by a rise of the rate of interest. The reverse happens when the marginal productivity of capital declines.

On the other hand, a decrease in the propensity to consume (or, in other words, an increase in the propensity to save) is accompanied by a fall of the rate of interest. For with a given initial income and a given rate of interest the expenditure on consumption is now lower. This causes the marginal efficiency curve in Fig. 2 to shift downard and a lower

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[1] If this process of adjustment involves a time lag of a certain kind a cyclical fluctuation, instead of equilibrium, is the result. Cf. Kalecki, "A Theory of the Business Cycle," Review of Economic Studies, February, 1937.

quantity of investment corresponds to any given rate of interest. Total income decreases both as a direct result of the decreased expenditure on consumption and because of the diminished quantity of investment. Thus a downard shift of the liquidity preference curve in Fig.1 takes place. The consequence is a fall of the rate of interest. In a similar way an increase in the propensity to consume raises the rate of interest.

Thus the two traditional statements that the rate of interest rises together with the marginal net productivity of capital, and vice versa, and that it moves in the opposite direction to the propensity to save, hold fully in our generalised theory. Two limiting cases, however, deserve special attention.

The theory put forward is quite general and formal. The actual reactions, however, depend on the concrete shape of the functions (1),(2) and (3). We are concerned at present with the consequences of different shapes of the liquidity preference function. For the general case it has been assumed that the demand for liquidity is a decreasing function of the rate of interest and an increasing function of total income. The demand for liquidity (i.e., for cash balances) has thus two elasticities: an interest-elasticity which is negative and an income-elasticity which is positive. These two elasticities determine the reaction of the rate of interest to changes in the marginal efficiency of investment (which is correlated to the marginal net productivity of capital) and in the propensity to consume; for the reaction of the rate of interest to these is due to the influence which the change of income caused by them exerts upon liquidity preference. The greater the income-elasticity of the demand for liquidity the more the curve of liquidity preference is shifted when income changes and, consequently, the greater is the reaction of the rate of interest. The shift of the liquidity preference curve changes the demand for liquidity corresponding to any given rate of interest. If, however, the amount of money (in wage-units) is fixed, the rate of interest must change so as to equalise the demand for liquidity to the quantity of money available. The change of the rate of interest which thus follows is the greater the smaller the interest-elasticity of the demand for liquidity. Therefore, the reaction of the rate of interest is the greater the smaller the interest-elasticity of the demand for liquidity. In the special case in which the income-elasticity of the demand for liquidity is zero the rate of interest does not react at all to changes other than in the quantity of money (measured in wage-units). The demand for liquidity is in this case a function of the rate of interest alone:

$$M = L(r) \tag{1a}$$

There is but one curve of liquidity preference and the amount of money determines the rate of interest independently of the level of total income. Changes in the marginal efficiency of investment and in the propensity to consume do not affect the rate of interest at all. The whole brunt of such changes has to be borne by the other variables of the system (i.e., expenditure on consumption, investment and income). The same result is also reached when the interest-elasticity of the demand for liquidity is infinite. In this case, too, the rate of interest does not react to changes in the marginal efficiency of investment or in the propensity to consume. For the change of the rate of interest which is necessary to balance a given change in the demand for liquidity caused by a change of total income is nil in this case. This is Mr. Keynes' theory. Since Mr. Keynes recognises expressis verbis the dependence of the demand for liquidity on total income[1] it is obviously the last case he must have in mind.

The other special case is when the interest-elasticity of the demand for liquidity is zero. The demand for cash balances is in this case a function of income alone:

$$M = L(Y)$$

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[1] Cf. The General Theory of Employment, etc., pp. 171-172 and pp. 199 et seq.

Both Y and M being measured in wage-units (or in any other numéraire, for instance, wheat[2]) This equation states simply the proportion of their real income people hold in cash (in real balances). If this proportion is regarded as constant our function becomes:

$$M = kY$$

(where k is a constant) which is the well known Cambridge equation of the quantity theory of money. Taking into account equation (5) this can be written $Q = kYw$, or $Q = wL(Y)$ in the more general case, where Q is the quantity of money and w is the money price of the commodity or service which has been chosen as numéraire. The latter being given, the total income is determined by the quantity of money. Total income being given, the rate of interest is determined exclusively by the equations (2), (3) and (4), i.e., by the propensity to consume, by the marginal efficiency of investment (which in turn depends on the marginal net productivity of capital), and by the condition that investment is equal to the excess of income over expenditure on consumption (i.e., saving). This is the traditional theory of interest.

Thus both the Keynesian and the traditional theory of interest are but two limiting cases of what may be regarded to be the general theory of interest.

5. It is a feature of great historical interest that the essentials of this general theory are contained already in the work of Walras.

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[2] The reader will be reminded that Marshall and Professor Pigou have used wheat as a numéraire in this connection. Vide Marshall, Money Credit and Commerce, p.44, and Pigou, Essays in Applied Economics, p.177.

Indeed, the demand for liquidity appears in Walras as the encaisse désirée. Walras is quite explicit about the fact that the demand for liquidity is a function of the rate of interest. This dependence is expressed as early as in the second edition of his Eléments d'économie politique pure which was published in 1889. "In a society--he writes--where money is kept in cash from the moment when it is received until the day when it is given into payment or loaned out, money renders few services and those who keep it, producers or consumers, lose needlessly the interest on the capital which it represents." ("Dans une société oú on garde la monnaie en caisse depuis le moment oú on la reçoit jusqu'au jour oú on la donne en paiement ou jusqu'au jour oú on la prête, la monnaie rend peu de services, et ceux qui la détiennent, producteurs ou consommateurs, perdent inutilement l'intérêt du capital qu'elle représente")[1] This is emphasised even more in his Théorie de la Monnaie where we read about the service yielded by a given encaisse monétaire: "its satisfaction is obtained at the price of interest and this is why the effective demand for money is a decreasing function of the rate of interest" ("sa satisfaction se paie au prix d'un intérêt et c'est pourquoi la demande effective de monnaie est une fonction décroissante du taux d'intérêt").[2] He goes on, to quote again from the second edition of the Eléments, saying: "Suppose that on a certain day the existing quantity of money Q_u has diminished or that the demand for cash H which represents the utility of money has increased... Equilibrium will be re-established on the next

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[1]P. 382.

[2]P. 95 of the reprint in Estudes d'économie politique appliquée (published in Lausanne in 1898). This passage does not occur in the original edition in form of a separate book which was published in 1886 (Lausanne).

day on the market at a new and higher rate of interest at which the demand for cash will be reduced." ("Supposons qu'un jour la quantité existante de monnaie Q_u ait diminué ou que l'encaisse désirée H représentant l'utilité de la monnaie ait augmenté... L'équilibre ne s'établirait, le lendemain, sur le marchpe, qu'á un nouveau taux d'intérêt plus élevé auquel l'encaisse desirée se reduirait.")[1]. Walras also uses the device of expressing the demand for cash balances in real terms. It is a certain real purchasing power over which the individual wants to have command and he expresses it in terms of a numéraire.[2] If H is the demmand for liquidity in terms of the numéraire chosen and Q_u is the amount of money in existence, then the price p_u of money in terms of the numéraire is determined by the equation $Q_u p_u = H$, which is analogous to the equation (5) above.[3] Walras fails, however, to indicate whether the encaisse désirée depends also on the level of real income. But whatever the shortcomings of his presentation, the liquidity preference function has been indicated clearly by Walras.

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[1] P.383. In the last editions of the Eléments the exposition, though put into mathematics, is somewhat obscure. Walras introduces also the question of liquidity (i.e., of stocks) in other commodities. Of each commodity a stock is kept qhich renders a "service d'approvisionnement " (service of storage). The rate of interest is the cost of this service. Cf.Eléments, 4th ed., 1900,pp.179, 298,303.

[2] Pp.377-78 of 2nd ed. and Théorie de la Monnaie (as reprinted in Etudes d'économie politique appliquée), p.95.

[3] P 378 and p.383 of 2nd ed.

Our remaining three equations are also contained in the system of Walras. There is, first of all, the propensity to save (instead of our propensity to consume). Saving is defined, as by Mr. Keynes, as the excess of income over consumption (l'excédent du revenu sur la consommation)[4]. Now this excess of income over consumption is conceived by Walras to be a function of both the rate of interest and income . He expresses the propensity to save by an equation and states explicity that this equation "gives the excess of income over consumption as a function of the prices of the productive sevices and of consumers' goods and of the rate of interest" ("donnant l'excédent du revenu sur la consommation en fonction des prix des services et des produits consommables et du revenu net").[1] By introducing the prices of all commodities he brings income indirectly into the equation expressing the propensity to save. His equation thus corresponds to our equation (2). As a counterpart to our investment function Walras has an equation which determines the total value of "capitaux neufs" produced. This value is determined by the condition that the selling price of the capitaux neufs (which is equal to the capitalised value of their net returns) is equal to their cost of production.[2] This equation determines the total volume of investment corresponding to any given rate of interest. Unfortunately, Walras fails to indicate on what the net return of the capitaux neufs depends. He takes it just for granted and as a consequence there is no relation between their net return and the expenditure on consumption.

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[4]P. 281 of first edition published in 1874 (p. 269 of second ed. and p. 249 of final ed.). Walras uses throughout the term excédent and the word épargne is reserved to denote net saving. Cf. p. 282 of first ed. (p. 270 of 2nd ed. and p. 250 of final ed.).

[1]P. 271 of 2nd ed. "Taux du revenu net" must be translated by "rate of interest" in this connotation.

[2]Cf. 284 of first ed. (pp. 246-7 and p. 253 of final ed.).

Finally Walras expresses in a separate equation the equality of the value of the capitaux neufs and the excess of income over consumption.[3] This, however, is not equivalent to our equation (4) which states the equality of investment and the excess of income over consumption. For there is an important difference. In our system, as in the theory of Mr. Keynes, equation (4) is an identity. Whatever the investment and saving decisions are, the volume of total income always adjusts itself so as to equalise saving and investment actually performed.[4] This is a simple budget relationship, for the individuals' incomes are equal to the sum of expenditure on consumption and investment. Walras, however, treats the equality of investment and saving not as an identity but as a genuine equation which holds true only in a position of equilibrium. Hence his investment (value of the capitaux neufs) and saving (excess of income over consumption) are to be interpreted as decisions which finally are brought into equilibrium by a change in the rate of interest and in total income.[1] But this equation does not show how total income changes so as to bring saving actually performed always into equality with investment.

This is done by our identity (4) which corresponds to the sum of the budget equations in the Walrasian system and shows how expenditure on consumption and investment determine the total income. When this budget relationship is taken account of, there is no need any more for a separate equation indicating the equilibrium of saving and investment decitions based on some given income, however defined. All the relevant relation are expressed by our equations (2), (3) and (4). Thus Mr. Keynes' apparatus involves a considerable simplification of the theory.

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[1]P.286-7 of 2nd ed. (pp.266-67 of final ed.). In the process of tatonnements described by Walras all the prices change and thus total income changes, too.

[3]P.284 of first ed. (p.252 of final ed.).

[4]It ought to be mentioned here that this has been recognised by many economists before Mr. Keynes. If investment decisions exceed saving decisions "forced saving" takes place according to a widely accepted doctrine. And Mr. Robertson has pointed out (cf.Money, London, 1928,pp.93-97) that if saving decisions exceed investment decisions the excess cannot be saved. It becomes "abortive".

6. Having investigated the consequences which the introduction of liquidity preference has for the formulation of the theory of interest, let us see how the general theory outlined above can be applied to solve the problem which is the concern of all theories of underconsumption. Mr. Keynes has scarcely done justice to what is the core of the argument of those theories. "Practically-he writes-I only differ from these schools of thought in thinking that they may lay a little too much emphasis on increased consumption at a time when there is still much social advantage to be obtained from increased investment. Theoretically, however, they are open to the criticism of neglecting the fact that there are two ways to expand output."[2] Mr. Keynes treats investment and expenditure on consumption as two independent quatities and thinks that total income can be increase indiscriminately by expanding either of them. But it is a commonplace which can be read in any textbook of economics that the demand for investment goods is derived from the demand for consumption goods. The real argument of the underconsumption theories is that investment depends on the expenditure on consumption and, therefore, cannot be increased without an adequate increase of the latter, at least in a capitalist economy where investment is done for profit.

Few underconsumption theorists ever maintain that any saving discourages investment.[1] Generally they maintain that up to a certain point saving encourages investment while it discourages it if this point is exceeded.[2] This is the theory of oversaving. If people would spend

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[2]The General Theory of Employment, etc., p.325.

[1]The most prominent among those who did so was Rosa Luxemburg in her famous book Die Akkumulation des Kapitals (Berlin, 1912).

[2]Vide, for instance, Hobson, The Industrial System, London, 1910, pp. 53-54.

their whole income on consumption, investment would obviously be zero, while the demand for investment would be zero too, if they consumed nothing. Thus mere common sense suggests that there must be somewhat in between an obtimum propensity to save which maximises investment. But no under consumption theorist ever has shown what this optimum is and how is is determined. The problem, however, was put forward with unsurpassed clarity already by Malthus: "No considerable and continued increase in wealth coud possibly take place without that degree of frugality which occasions, annually, the conversion of some revenue into capital, and creates a balance of produce over consumption; but it is quite obvious . . . that the principle of saving, pushed to excess, would destroy the motive to production. . . If consumption exceeds production, the capital of the country must be diminished, and its wealth must be gradually destroyed from its want of power to produce; if production be in great excess above consumption, the motive to accumulate and consume must cease from the want of will to consume. The two extrems are obvious; and it follows that there must be some intermediate point, though the resources of political economy may not be able to ascertain it, where taking into consideration both the power to produce and the will to consume, the encouragement to the increase of wealth is greatest."[1]

The general theory of interest outlined in this paper enables us to solve this problem and to determine the optimum propensity to save which maximises investment. Since investment per unit of time is a function of both the rate of interest and expenditure on consumption a decrease of the propensity to consume (increase in the propensity to

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[1] Principles of Political Economy, London, 1820, pp.8-9(introduction). Cf. also pp. 369-70.

save) has a twofold effect. On the one hand the decrease of expen- on consumption discourages investment, but the decrease in the propensity to consume also causes, as we have seen, a fall of the rate of interest which encourages investment on the other hand. The optimum propensity to consume is that at which the ecouraging and the discouraging effect of a change are in balance.

The condition of such a balance is easily found. A change of the propensity to consume is mathematically a change of the form of the function (2) in our equations. We want to discover the conditions this function has to satisfy in order to maximise investment. Let δC be the variation of expenditure on consumption and δi the variation of the rate of interest which are caused by the change of the propensity to consume. Recalling the investment function (3), which is $I = F(i, C)$, the condition that investment be a maximum is then:

$$\delta I = F_i \delta i + F_c \delta C = 0 \qquad (6)$$

where δI is the corresponding variation of investment.

From equation (4) we derive the variation of total income caused by the change of the propensity to consume:

$$\delta Y = \delta C + \delta I$$

and since $\delta I = 0$ when investment is a maximum we have in the maximum position:

$$\delta Y = \delta C \qquad (7)$$

Now the change of the rate of interest due to the change of the propensity to consume can be obtained from equation (1), i.e., from the liquidity preference function. We have:[1]

$$\delta M = L_i \delta i + L_Y \delta Y \tag{8}$$

If the sum of real balances available, i.e., the quantity of money measured in wage-units or in any other numéraire, is assumed to be constant[2] this reduces to:

$$L_i \delta i + L_Y \delta Y = 0 \tag{8a}$$

whence:

$$\delta i = -\frac{L_Y}{L_i} \delta Y \tag{9}$$

By substitution of (9) and (7) in (6) we arrive at the equation:

$$-F_i \frac{L_Y}{L_i} \delta C + F_c \delta C = 0$$

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[1]The liquidity preference holds only for a given distribution of incomes (cf. Footnote 1 on p.170 above). Similarly the investment function holds only for a given distribution of the expenditure for consumption between the different industries, for even if the total expenditure on consumption remains unchanged a shift of expenditure from goods requiring less to goods requiring more capital to produce, or vice versa, necessarily affects investment. Equations (6) and (8) in the text presuppose, therefore, that changes in the distribution of incomes and in the direction of consumers' expenditure to different industries are either absent, or that their effect on total investment and on the total demand for liquidity is of second order magnitude and can thus be neglected. Since a change in the distribution of incomes and of consumers' expenditure the second assumption is the only realistic one. A more precise theory would have to take into account the effect of these changes, too.

[2]If the money wage (or, more generally, the money price of the numéraire chosen) is constant, this means that the nominal quantity of money is constant, too. If not, the nominal quantity of money has to change proportionally to the money price of the numéraire. If, however, labour is not regarded as a homogeneous factor the use of labour-units as numéraire involves really the use of a particular index number, i.e., the labour standard, and our assumption amounts to assuming that the puchasing power of money in terms of the labour standard is constant.

which can be transformed into:

$$-\frac{L_Y}{L_i} = -\frac{F_c}{F_i} \qquad (10)$$

This equation, together with the equations (1), (3) and (4) of our system, determines the optimum propensity to consume under the assumption that the amount of money (measured in wage-units) is constant.[1]

Only such forms of the function representing the propensity to consume which satisfy this equation provide a maximum investment. A very simple economic interpretation can be given to the equation obtained. The right hand side of the equation is the marginal rate of substitution between a change of the rate of interest

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[1] If the amount of money (as defined in the text) is allowed to change a more general condition is obtained. For this purpose we must add to our system of equations a supply function of money. Let this function be:

$$M = \psi(i, Y)$$

where M and Y are measured in terms of wage-units. We have then:

$$\delta M = \psi_i \delta i + \psi_Y \delta Y$$

and taking into account equation(8) in the text we obtain:

$$\psi_i \delta i + \psi_Y \delta Y = L_i \delta i + L_Y \delta Y$$

which can be written in the more convenient form:

$$(\psi_Y - L_Y)\delta Y = (L_i - \psi_i)\delta i$$

whence we get:

$$\delta i = \frac{\psi_Y - L_Y}{L_i - \psi_i}\delta Y$$

Substituting this and (7) in (6) we arrive at:

$$F_i \frac{\psi_Y - L_Y}{L_i - \psi_i}\delta C + F_c \delta C = 0$$

which is, finally, transformed into:

$$\frac{\psi_Y - L_Y}{L_i - \psi_i} = -\frac{F_c}{F_i} \qquad (10a)$$

This is the most general form of the equation which determines the optimum propensity to consume. Equation (10) obtained in the text is a special case of it when $\psi_Y = 0$ and $\psi_i = 0$.

and a change of the expenditure on consumption as inducements to invest. The left hand side is the marginal rate of substitution between a change of the rate of interest and a change of real income as determining the demand for liquidity. The optimum propensity to consume is thus determined by the condition that the marginal rate of substitution between the rate of interest and total income as affecting the demand for liquidity is equal to the marginal rate of substitution between the rate of interest and expenditure on consumption as inducements to invest.[1]

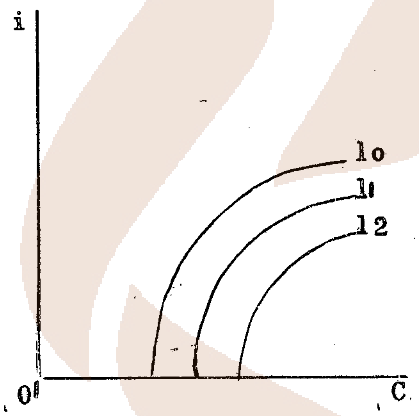

Fig.4.

It is convenient to have a graphic illustration of this condition. On Fig. 4 we draw a family of indifference curves indicating the possible variations of the rate of interest and of the expenditure on consumption which do not change the level of investment per unit of time. We may call them isoinvest-

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[1]The economic interpretation of equation (10a) is similar to that of equation (10), only the left hand side is here the marginal rate of substitution not along a curve of equal liquidity (isoliquidity curve; vide below) but along the curve corresponding to the equation:

$$\psi(i,Y) = L(i,Y)$$

Thus the left hand side of (10a) is the marginal rate of substitution between the changes of the rate of interest and of total income which are compatible with the maintenance of the equality of the supply of and the demand for money. The supply function of money depends on the behaviour of the monetary system.

ment curves. The expenditure on consumption being measured along the axis O'C and the rate of interest along O'i these curves slope up-ward[2] and the greater the level of investment the more to the right is the position of the corresponding isoinvestment curve.[1] The curves can be expected to be concave downwards, for the stimulus to invest exercised by each successive increment of expenditure on consumption is weaker. This is explained by the increasing prices of the factors of production which diminish the net return derived by entrepreneurs from successive increments of expenditure on consumption (the curves of marginal efficiency of investment in Fig. 3 are shifted upwards less and less). Thus the greater the expenditure on consumption the greater is the increment of it which is necessary to compensate a given rise of the rate of interest. Finally, we reach a point where a further increase of the expenditure on consumption fails entirely to stimulate investment. This happens when the elasticity of supply of the factors of production has become zero, so that an increase of the expenditure on consumption only raises their prices. Thus the isoinvestment curves become horizontal

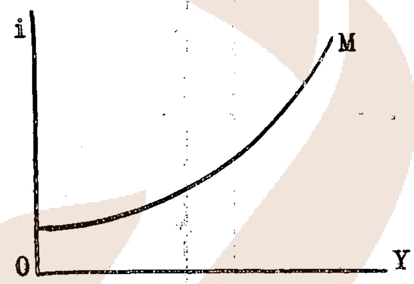

Fig. 5

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[2]The slope of these curves is $-\frac{F_c}{F_i}$. Since $F_c > 0$ and $F_i < 0$ the slope is positive.

[1]There are certain combinations of the expenditure on consumption and of the rate of interest at which the existing capital is just maintained by replacement. They determine the curve corresponding to zero investment (i.e., the curve I_0 in Fig. 4). All curves to the right of it correspond to positive and all to the left correspond to negative investment.

to the right of a certain critical value of the absicssa.[2]

On Fig. 5 we draw an indifference curve which represents all the variations of the rate of interest and of total income which do not affect the demand for liquidity (total income and the demand for liquidity being expressed in wage-units). We may call it the isoliquidity curve. Since the amount of money is assumed to be given we have only one such curve (the curve M in Fig. 5). It slopes upward[1] and is straight, convex or concave downard, according as the demand for liquidity increases with an increase of real income at a constant, an increasing or a decreasing rate, respectively.[2] Downard convexity, however, seems to be the case which is practically most likely to occur.[3]

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2. $-\frac{F_c}{F_i} = 0$ when $F_c = 0$.

1 The slope of the curve is $-\frac{L_Y}{L_i}$. It is positive because $L_Y > 0$ and $L_i < 0$. In the limiting cases, however, where either $L_Y = 0$ or $L_i = 0$ we have either $-\frac{L_Y}{L_i} = 0$ or $-\frac{L_Y}{L_i} = \infty$ and the isoliquidity curve degenerates into a horizontal or vertical straigt line.

2 We have $\frac{d^2i}{dY^2} = -\frac{L_{YY}L_i - L_{iY}L_Y}{L_i^2}$. Taking $L_{iY} = 0$ approximately and remembering that $L_i < 0$, we find that $\frac{d^2i}{dY^2}$ is of the same sign as L_{YY}. (Footnote added, 1943).

3 The last two sentences were revised by the author, 1943.

The optimum propensity to consume can now be determined in a simple way by combining the diagrams of Fig. 4 and Fig. 5. Equation (10) states that the slope of the slope of the isoliquidity curve has to be equal to the slope of the isoinvestment curve (vide the point P in Fig. 6). But the position of origins 0 and 0' in the combined diagram is not arbitrary. For 00' is the difference between total income and expenditure on consumption, i.e represents the level of investment. Thus to each level of investment there belongs a special length of 00'. The optimum propensity to consume is, therefore, obtained by super-imposing Fig. 5 upon Fig. 4 (as in Fig. 6) and moving it horizontally until the isoliquidity curve becomes tangent to the isoinvestment curve whose index (i.e., level of investment) is equal to the length of 00'. 00' is then the maximum investment, 0'A and 0A are the expenditure on consumption and the total income which correspond to it. The isoinvestment curves[1] being concave downard, an optimum propensity to consume exists an is unique if the isoliquidity is convex downard or is a straight line, or even if it is concave downard, provided its concavity is less than that of the isoinvestment curves and its curv-

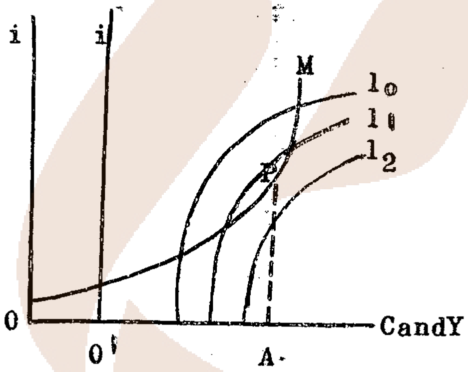

Fig. 6

[1]Balance of this sentence was revised by the author, 1943.

ature does not change sign.[2]

From Fig.6 we obtain the expenditure on consumption 0'A and the total income 0A which correspond to maximum investment and which are, as we have seen, uniquely determined. Plotting them on a diagram (vide Fig.7) we obtain a point R through which the curve representing the propensity to consume has to pass. Thus the function expressing the optimum propensity to consume is determined only by one point through which it has to pass. Any function which passes through the point R maximises investment. Any function, however, which does not pass through R makes total investment smaller. Generally we may expect that a de-

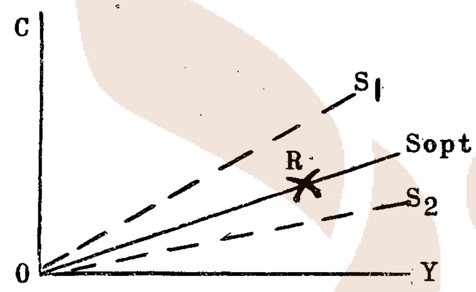

Fig. 7

crease of the propensity to consume (i.e., an increase of the propensity to save) leads us from curves which pass above R to curves which pass below R (e.g., the curves S_1 and S_2 in Fig. 7). As long as they pass above R the propensity to consume is above optimum, when they

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[2] The graphic solution indicated in Fig. 6 is also applicable to the general case where the quantity of money (in terms of wage-units) is not constant. As shown in the footnotes 1 on pp.185 the equation (10a) is substituted in this case for the equation (10). Instead of the isoliquidity curve we get a curve corresponding to the equation $\psi(i,Y) = L(i,Y)$. It is a projection on the Yi plane of the curve resulting from the intersection of the two surfaces representing the supply and the demand for money respectively (the isoliquidity curves are a special case of it obtained when the supply surface of money is a plane parallel to the Yi plane). The shape of the curve depends now also on the form of the supply function of money. The graphic solution is obtained as in the text by moving the diagram of this curve horizontally until the curve becomes tangent to the isoinvestment curve corresponding to the level of investment equal to 00'.

pass below R it is below optimum. Maximum investment is attained when we hit upon a curve which passes through R (e.g., the curve Sopt in Fig.7). This is a curve of optimum propensity to consume. Any change of the shape of the curve which does not affect its passing through R is irrelevant.

7. Let us now apply the result obtained to two special cases. When either the income-elasticity of the demand for liquidity is zero or the interest-elasticity of the demand for liquidity is infinite, which is the case corresponding to Mr. Keynes' theory, we have either $L_Y = 0$ (and $L_i \neq 0$) or $L_i = \infty$ (and $L_Y \neq 0$). It follows immediately from equation (10) that $F_c = 0$ in either case.[1] The economic interpretation is simple. As we have seen, in this case a change in the propensity to consume does not affect the rate of interest at all. The rate of interest remaining constant, the optimum propensity to consume is when the expenditure on consumption is such that a further increase does not any more increase the marginal efficiency of investment. It has been mentioned already that this happens when the elasticity of supply of factors of production becomes zero, so that an increase of the expenditure on consumption only raises their prices but cannot increase investment. This implies the absence of even voluntary unemployment of factors of production. If involuntary unemployment of a factor is defined by its supply being infinitely elastic, it is absent whenever the elasticity of supply is finite. A zero elasticity of supply, however, means that there are no more factors which would offer their services if the remuneration were greater, i.e., are voluntarily unemployed. Until this stage is reached any increase in the propensity

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[1] It seems, however, highly doubtful that $L_i = \infty$ over the whole range of the liquidity preference function.

to consume stimulates investment.[2] This fits well into the scheme of Mr. Keynes' theory.

The other special case is when the interest-elasticity of the demand for liquidity is zero which is, as we have seen, the case of the traditional theory. Then $L_i = 0$ (and $L_Y \neq 0$) and by rewriting equation (10) in the form:

$$-\frac{L_i}{L_Y} = -\frac{F_i}{F_c}$$

we obtain $F_i = 0$ for this case. Any decrease in the propensity to consume stimulates investment by causing an appropriate fall of the rate of interest. The propensity to save can never be excessive, for the rate of interest falls always sufficiently to make room for additional investment. The only limit is when a further decrease of the rate of interest stops increasing investment ($F_i = 0$), i.e., when the net return on investment becomes zero and the rate of interest is zero, too.

In the general case the optimum propensity to save is somewhere between these two limits and it is the greater the greater the income-elasticity and the smaller the interest-elasticity of the demand for liquidity. For the fall of the rate of interest due to an increase in the propensity to save is the greater the greater is the first and the smaller is the second of these two elasticities. The optimum propens-

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[2] In the general case where the quantity of money is allowed to vary the same result is reached when $\mathcal{L}_Y = L_Y$ (vide equation (10a)). In this case the income elasticity of supply of money is equal to the income-elasticity of supply of the demand for liquidity; each change of total income is balanced by exactly such a change of the supply of money that the rate of interest remains constant.

ity to save is also the greater the greater the elasticity of investment with respect to the rate of interest and the smaller the elasticity of investment with respect to expenditure on consumption.

Thus we arrive at the result that, with the exception of the special case covered by the traditional theory of interest, there exists an optimum propensity to save[1] which depends of the shape of the liquidity preference and of the investment functions. This imposes a maximum limit on investment per unit of time and any attempt to exceed it by raising the propensity to save above its optimum frustrates itself by leading to a diminution of investment. In a society where the propensity to save is determined by the individuals there are no forces at work which keep it automatically at its optimum and it is well possible, as the underconsumption theorists maintain, that there is a tendency to exceed it. Whether this is actually the case is a matter for empirical investigation and cannot be answered by the economic theorist.

The optimum propensity to save is, however, defined only with regard to a given quantity (or more generally: to a given supply func-

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[1] "Optimum" means here merely "maximising investment." This need not be the most desirable propensity to save from the point of view of social policy. From the latter point of view a propensity to save which maximises real income may be more desirable. My "optimum" propensity to save, however, maximises the speed of growth of wealth. (Footnote added in 1943.)

tion) of money. Therefore, if the propensity to save does exceed its optimum it need not be curbed to avoid its evil consequences. It can be made to benefit economic progress by an appropriate monetary policy which increases the quantity of money sufficiently to reduce the rate of interest so as to compensate the discouraging effect a high propensity to save has on investment.[1] How far such a policy is possible depends on the structure of the monetary and of the whole economic system.

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[1] The requirement of an increase of the quantity of money to counteract an excessive propensity to save is not in contradiction with the teaching of Professor Davidson, Professor Hayek and Mr. Robertson that technical progress does not require an increase of the quantity of money to avoid deflation. If the increase in the propensity to save is accompanied by technical progress which increases the marginal efficiency of investment, investment is not discouraged and no increase of the quantity of money is necessary.

LRL.