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A MODEL FOR SIMULATION OF AN ECONOMY

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## A MATHEMATICAL MODEL OF AN ECONOMY

The deficiencies of economic models are well known: either they are very aggregative and their few variables do not permit describing the economic phenomena with sufficient detail, or they contain unacceptable hypotheses of linearity, or they are obtained by statistical fitting using variables for which the statistics exist but which never permit reproducing the causal relations satisfactorily. Moreover, they do not permit studying the influence of extra-economic factors.

Hardly ever are they very susceptible of improvement: each change means beginning the whole job over and, in cases in which mathematical solutions in terms of explicit formulas are sought, the introduction of small modifications in the relations used can fundamentally change the order of difficulty of the problem.

The method of simulation, or numerical experimentation as some prefer to call it, seeks to overcome those difficulties by dint of minutely detailed work and the use of many hours of electronic computation (which is not cheap).

In principle, the method permits introducing, with all the necessary flexibility, all the variables that appear important or interesting for describing the economy. In practice, this has two limitations: a) the proliferation of variables--each one of which has its own evolution in time--can confuse more than it clarifies: all analysis must have a limit in order to be useful; b) the capacity of modern computers is not unlimited and can be

surpassed if one is too ambitious. Moreover, it is important to point out that the more variables there are, the more data is needed, and that adds to the cost of the model in time and money.

What a simulation model tells us--like any other kind of dynamic model--is how the economic variables evolve during a certain period (it is anticipated for 10 or 15 years), if the hypotheses of the models are fulfilled.

These hypotheses are of four kinds:

- a) Values of the variables at the beginning of the "numerical experiment" (initial values).
- b) Values of the parameters which appear in the relationships.
- c) Logical or mathematical form of the relations between the variables.
- d) The surrounding conditions.

These four types of hypotheses have diminishing reliability. The initial values are obtained directly from the statistics, if they exist. The parameters are obtained by well known statistical methods: it is the central problem of econometrics. On the other hand, the form in which variables are related in economics is much more subject to argument than in physics or chemistry. As for the conditions of the social and political environment, it can even be argued whether they are susceptible to precise definition. For example:

How does the consumption of families depend on disposable income,  $YD$ ? Is it a linear function, or does  $\sqrt{YD}$  or  $\sqrt[3]{YD}$  or  $\log YD$  come into it? Furthermore, is it likely that  $YD$  is the only factor which determines consumption? Do not the previous habits of consumption, the interest on time payments, the rate of inflation, etc., also have <sup>appreciable</sup> influence? And if so, how does one express this influence? And what are the socio-psychological motivations for consuming? Can the government guide consumption?

The specified social conditions, which will serve as a frame for the model (and which can vary in time and from one case to another), should determine the character of global consumption. After much discussion, perhaps the form  $C = a + b\sqrt{YD}$  is adopted, and surely there will not be unanimity. But once that formula is adopted, it is easier to reach an agreement on the values of a and b and, much easier still, to know how much YD and C are in the initial year of the model.

To construct a dynamic model means to assign by hypothesis causal relations among the variables, in a way that, if values of the parameters are also given, one can calculate step by step (for example, each 20 days) the evolution in time of each variable. The calculations made, one has the history of that economic system for 10 years. The surrounding conditions are represented by using special forms for those relations or by means of exogenous variables, whose law of evolution during those 10 years is not determined (or only partially) by the model, but is also determined by explicit hypotheses.

In this way, one obtains, then, only one particular case: if some country should exist whose variables fulfilled those inter-relations and with those initial values and parameters, its history could not be other than that, provided the exogenous hypotheses are also fulfilled.

This is valid for any dynamic model, including that of Harrod-Domar. Therefore, up to this point the advantage of introducing electronic computation would consist only in allowing calculation of large, detailed, and complicated models. This advantage would be rapidly counterbalanced by the slight reliability of data and relations which becomes apparent as one analyzes an economic system in some detail.

The true advantage consists in that, thanks to computers, it is possible to repeat the calculation many times in a short period, each time introducing changes in the values, in the form of the relations, or in the exogenous hypotheses. Each calculation is a numerical experiment, carried out under different conditions. The number of experiments can be in the thousands. If after that some general conclusions cannot be extracted, it can be concluded that the blame is on the experimenter, not the method.

An example: suppose that one wishes to verify the exactness of a statement such as "a decrease of 4% in the consumption of the upper classes would permit the financing of a growth of 3% annually." A model then would be constructed where these variables would appear and all the related ones that must be taken into

account in the judgment of the model maker. The carrying out of numerous tests with this model, using different exogenous hypotheses and different parameter values considered "reasonable", would permit arriving quickly at some conclusions. If the national product never behaves as predicted, the statement is at least risky and the policy should not be applied without great precautions. If, on the other hand, it always behaves thus, the statement acquires reliability. The next step would be to test with other models, proposed perhaps by other economists, where the relations between the variables are not exactly the same, or new factors previously ignored appear. If, on trying these new models, the statement continues to be confirmed, it will begin to enter the category of natural law in which one will have to believe more as more experiments confirm it.

Most probably the statement will be confirmed by some tests and not by others. The analysis of the differences among those experiments will make it possible in principle to determine under which explicit hypotheses it is confirmed. Thus, one will know what additional measures it is necessary to take in order that that policy will be useful.

Stated in another way: everyone has in mind a model of reality when he tries to foresee the results of a proposed measure. Each effect that is predicted is based on certain hypotheses about cause-and-effect relations among the economic variables. In the mind of the layman, those hypotheses are very blurred and are presented one by one, rapidly losing their guiding thread. The cross

effects cannot be and are not taken into account. An economist is capable of seeing with more clarity each step of the reasoning, but there is no human mind that can follow the network of the lateral, mutual and sequential effects produced by a single initial measure.

Simulation makes it possible to clear up this confusing mental picture, focusing on its parts one by one in order to give it an explicit form and taking advantage for this purpose of the experience of all the economists that one wishes.

In this way, all the hypotheses that operate more or less obscurely in the reasoning of each individual are formulated explicitly. This obligation of stating clearly what are the basic propositions, not in general but in concrete applications, is perhaps one of the greatest practical merits of this technique.

It is often said that it is not possible to apply mathematical methods when there is no clear correlation among the variables which permits joining them in an unquestionable formula. This comes from an ancient conception of mathematics which identifies it with infinitesimal calculus or with algebraic equations or inequalities. Applied mathematics of today permits the exact translation of any type of reasoning, keeping its elements of uncertainty if necessary. It is a contradiction to state that one can make deductions that cannot be expressed mathematically unless by deducing or reasoning we mean operations that are not communicable to other human beings. But there we cross the dividing line between science and fiction.

If the economist has at hand a hypothesis about the connections between certain variables that can be understood by his colleagues, it can be expressed by logical-mathematical relations. The true difficulty is that, in general, different hypotheses exist simultaneously among which it is not known which to choose and, in consequence, in the usual "literary" reasoning, they are hopelessly mixed. They can now be expressed and experimented on separately.

At the risk of stating trivialities, it should be pointed out that if an economist says: "I believe that the consumption function is F, but only if there is social tranquility, while in case of disturbances I am inclined to G," there are not two hypotheses but only one. However it is, the economist must define clearly what he is going to call "tranquility" and "disturbance", so that the model can tell in each instance which of the two situations reign and apply then the corresponding function.

If one is not capable of defining those terms practically then what one says lacks meaning. It should be pointed out that, although the ideal would be to be able to define a variable, "level of tranquility", that would depend only on other variables of the model; it is not inconvenient if that dependence is only partial or if it is a question of a totally exogenous hypothesis. Moreover, the term "variable" does not have to be interpreted as a real continuous variable. It may be that this variable permits only two values: tranquility or non-tranquility, or a half dozen.



values if it is thought necessary. These hypotheses constitute the model of the economist.

Another important advantage of the simulation model is that it is not indispensable to formulate each hypothesis by a formula in the usual sense. It can have the form of simple logical alternatives, as for example: if unemployment is greater than so much and the balance of payments less than so much, foreign investments will be reduced by so much. It should be recognized that computers permit instructions of this kind, complete with the remaining alternatives.

In our opinion, then, it is not possible to be against the use of models, since there is no other way of thinking about the future. One can, of course, criticize particular models for being inadequate as one criticizes so many men for their inadequate image of reality. But nothing prevents adding factors to a model, correcting its defects, improving its relations, in the light of all the existing knowledge on the subject and arriving thus at an image of reality as clear and as complete as possible. Until now, such a task was useless, since there was no way of making the model "function": the mathematical tools only permitted solving very simple, inadequate cases. That is the bottleneck which computers have overcome.

Let us emphasize, finally, that the simulation models do not serve to optimize in the sense of linear programming and other techniques of operations research. Numerical experiments are limited to describing what would be the result of taking certain

economic measures, under certain hypotheses. One can then choose the policy which appears best among all the ones tried, but there is not any assurance that it will be "the best possible". The selection of policies to try must not be made then with very restricted criteria.

This does not mean that the techniques of optimization are preferable. They only serve when the objective function is scalar, which is generally an inadmissible simplification. He who believes that the state of the country's economy can be measured by one number--per capita national product, or any other--can optimize. Those of us who believe that the economy is described by a vector function, in which the national product is only one component, cannot even define an optimum unanimously. And when we speak of choosing the best policy among the ones tried, we refer to the value criterion of the person or institution that chooses. The same simulation model then can be useful to persons of different normative criteria; the evaluation is a posteriori. Models of optimization have the value judgment built in a priori, and for that reason must use extremely simplified criteria.