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THE FACTOR-PROPORTIONS PROBLEM IN UNDER-DEVELOPED AREAS *

by R.S Eckaus.

THE CONCEPTS 'STRUCTURAL DISEQUILIBRIUM' , 'OVERPOPULATION', 'TECHnological unemployment' and 'underemployment' appear frequently in the literature on underdeveloped areas and there is considerable discussion of the comparative desirability for use in such areas of relatively labour-intensive or capital-intensive techniques. This paper is intended to help clarify some of the underlying issues and to begin to provide a theoretical basis for their analysis.

Many of the underdeveloped areas of the world have large agrarian populations in which there is either persistent open unemployment or in which the marginal productivity of the working force is so low that it is commonly believed that withdrawal of a sizable fraction would not significantly affect output. This seems to be the case to varying degrees for much of Asia and the Middle East. Other countries, such as Italy, show persistent urban as well as rural unemployment or underemployment. It is a common feature of the unemployment in these countries that it fails to respond to fiscal policy measures designed to increase employment by stimulating effective demand. Use of conventional income-generating techniques appears in fact to create inflationary pressures and balance-of-payments difficulties long before full

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* The American Economic Review, September 1955, Reprinted by permission of the American Economic Association and the author. The author wishes to express his gratitude to the Center for International Studies, under whose auspices he did the work upon which this paper is based, to the seminar group of the Center for criticism and encouragement and especially to F.M. Bator and P.N. Rosenstein-Rodan. He is also indebted to P.A. Samuelson and R. Solow for their interest. An Italian translation of this article appeared in L'Industria, November 1955.

employment is approached.

This interpretation of the condition of many underdeveloped areas has led to the formulation of a number of alternative explanatory hypotheses which are presented in Section I. One of these hypotheses appears, at this stage of investigation, to be particularly fruitful in casting light on some of the outstanding characteristics of underdeveloped areas and is elaborated in Section II. Two approaches to the problems of empirical testing of the hypotheses are outlined in Section III.

The hypotheses presented below suggest that the unemployment difficulties of underdeveloped areas are not basically due to lack of effective demand but stem from market imperfections', limited opportunities for technical substitution of factors and inappropriate factor endowments.[1] The techniques of analysis of factor-market imperfections are well known.[2] The implications of limited technical substitutability of factors were first analysed by Abraham Wald[3] and more recently by the linear programming techniques.[4] Further development of the theoretical analysis in this paper consists mainly of an elaboration of geometrical techniques, which are used to apply the theory specifically to the problems of underdeveloped areas.

I. THE FACTOR-PROPORTIONS HYPOTHESES

The analysis which follows has grown out of the suggestion by C.P. Kindleberger that underdeveloped areas such as Italy are characterized by 'structural disequilibrium at the factor level'. This concept, formulated by Kindleberger and E. Despres, is identified as follows:

1/ The hypotheses and analysis have come to be known at the Center for International Studies, Massachussetts Institute of Technology, as the 'factor-proportions' problem.

2/ E.g., Joan Robinson, Essays in the Theory of Employment, 2nd ed. Oxford, 1947), ch. 2.

3/ A. Wald, 'Uber einige Gleichungssysteme der mathematischen Okonomie', Zeitschr. f. Nationalokon., Dec. 1936, VII, 636-70; cf. also W.L. Valk, Production, Pricing and Employment in the Static State (London, 1937), p.58.

4/ E.g., R. Dorfman, Application of Linear Programming to the Theory of the Firm (Berkeley, U.S.A., 1951). In his paper 'Full Employment and Fixed Coefficients of Production' (Quarterly Journal of Economics, Feb.1955, pp. 23-44), M. Fukuoka also relates the assumption of fixed coefficients in production to the problem of unemployment.

> Disequilibrium at the factor level may arise either because a single factor receives different returns in different uses or because the price relationships among factors are out of line with factor availabilities.[1]

This suggestion has been the starting point for two types of explanation of unemployment or underemployment in underdeveloped areas. The first type assumes that available technology would permit full use of the working force at some set of relative prices and finds the source of unemployment in various types of 'imperfections' in the price system. The second type suggests that there are limitations in the existing technology or the structure of demand which lead to a redundancy of labour in densely populated, underdeveloped areas. The two types of hypotheses are combined in Section 11 to obtain a more general analysis.

The Market Imperfections Hypotheses

In the accompanying figure the vertical axis represents the rate of real wages and the horizontal axis the amount of labour. The curves DD' and SS' represent the aggregate supply and demand relations for a typical industry if factor markets are competitive. Under competitive conditions the wage rate would settle at E.[2]

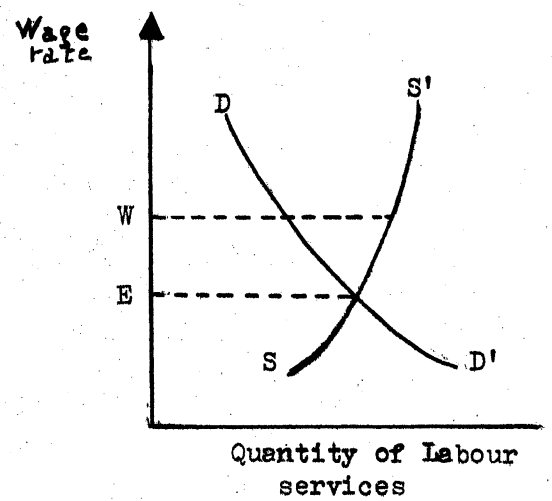

Figura I

1/ C.P. Kindleberger and E. Despres, 'The Mechanism for Adjustment in International Payments- The Lessons of Post-war Experience', The American Economic Review, Proceedings, May 1952.

2/ Fixed supply and demand curves such as those in Figure 1, suppose, of course, constant resources, technology and consumer tastes.

Suppose, however, that trade union pressures, immobility of labour, government social legislation or other factor-market imperfections maintain the wage rate at W rather than allowing it to fall to E. The effective labour supply curve would be WS'. At the higher wage rate the demand for labour would not absorb all the labour available and it could be said, as Kindleberger does, that, ceteris paribus, the wage rate does not represent factor endowments.

To isolate the influence of various types of imperfections let us now consider a case in which factor mobility, or lack of it, is not important and continue to confine the analysis to a closed economy. If the system had become adjusted to a particular complex of rigidities there would be no need for factor mobility in the absence of changes in techniques or tastes.[1]

The comparative use of the factors of production, depending as it does on the factor-price ratios and technology, would, however, reflect the 'true availability' of labour only if wages were kept at E in Figure I. If wages are kept at W there is an 'artificially' high ratio of the price of labour to the price of capital. Since we are explicitly assuming that factor substitution is, in fact, possible, a structure of production may result with a higher capital-labour ratio than otherwise. If the diagram were representative of large parts of an economy, as output increased full employment of the given labour force would require the use of more capital than if the structure of production were adjusted to a lower labour-capital price ratio, unless the substitution effects were offset by increasing returns to scale. In a country in which capital was scarce and unemployment of considerable magnitude, the attempt to achieve full employment by use of relatively capital-intensive investment would be more likely to lead to inflation and balance-of payments difficulties, short of full employment, than if more labour-intensive techniques were used.

The development of social policy in economically underdeveloped areas

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1 / 'If effective demand always moved up and down in the same well-worn channels, a supply of each type of labour would always be ready waiting to meet demand for it, when effective demand expanded, and the question of mobility would not arise.' Joan Robinson, op.cit., p. 30.

frequently proceeds more rapidly than economic growth. Imitation of the techniques of more advanced countries is not confined to technology. Elaborate social security legislation and aggressive government-encouraged union movements are often found in densely populated, low per capita income countries which are just on the threshold of economic advancement. There is little or no scope for such devices for raising wages except in the relatively more advanced and well organized sectors. Therefore, new industrial projects may face the prospects of wage rates quite different from those prevailing in the handicraft and agrarian sectors and thus may be compelled to use different factor proportions. These considerations suggest that the foregoing analysis may be quite relevant for underdeveloped areas.

The next step in the analysis is to abandon the assumptions of constant technology and consumer tastes and to investigate the effects of changes in the composition of demand for goods and factors due to such influences as changes in methods of production or in the directions or levels of demand as a result of changes in tastes or foreign competition. In this second case, as the level of aggregate effective demand rises, goods will be demanded in different proportions than formerly and the location and magnitudes of the demand for factors of production will shift. If labour is not mobile, or if it takes considerable wage increases to shift it, then factor disequilibrium such as depicted in Figure 1 for the preceding case of constant tastes and technology would develop in certain industries. An increase in the level of effective demand would push other industries to the limits of capacity relatively quickly in this second case. Money wages and prices would begin to rise, not uniformly but in the 'bottleneck' sectors, prior to the achievement of general full employment. New investment in these sections would tend to increase still further the substitution of capital labour while offsetting tendencies in the relatively stagnant sectors would work slowly, if at all. The balance of payments under the pressure of growing domestic inflationary pressures and increased demand for capital imports would tend also to develop deficits at an earlier stage in the expansion of national income. This could all be superimposed upon and could aggravate the 'factor disequilibrium' previously discussed. It would be distinguishable, however, as there would be evidence of excess

capacity and stranded capital-goods resources indicating an original misallocation or a structural shift.

There is at least superficial evidence to suggest that the factors stressed in this hypothesis may be operative in some underdeveloped countries. For example, although Italy has a persistent unemployment of about 2 million in a population of around 47 million, there are also some sectors of the Italian economy, such as shipbuilding, in which there is persistent unused physical capital plant and equipment. Moreover, we would expect that in underdeveloped areas the working force would be even more bound by tradition, reluctance to change location and barriers to social as well as physical movement than is the case in more advanced, industrialized countries: this would also contribute to the problems created by structural change.

Closely related to this second hypothesis is an explanation which locates the source of factor disequilibrium in barriers to the entry of new firms into profitable industries whose expansion is limited by various types of monopolistic restrictions. This and the other types of 'imperfections' could aggravate the 'factor disequilibrium'.

One further related hypothesis remains to be considered here.[1] Suppose that, whatever the actual characteristics of the production function and degree of technical substitutability of factors, businessmen believe that they face a production function with constant coefficients, i.e., no factor substitution is possible. Indian businessmen, for example, may believe that the 'American way' of producing is the best and only way and that this always involves high ratios of capital to labour. Plant engineers accustomed to emulating 'Western' technology may not be sensitive to the range of choice actually available in manufacturing processes and may impose unnecessary technical constraints on managers in underdeveloped countries. Thus in Figure 2, although the solid lines, x_1, x_2... may represent the real contours of the production function, businessmen may regard the dashed lines x'_1, x'_2... as the ones along which they must move.

In this case the expansion path P would be independent of the factor-price ratios, and, therefore, of the supply curve of labour such as indicated

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1 / I am indebted to F. M. Bator for the suggestion of this case.

in Figure 1. Expansion of effective demand would tend to run into the limits imposed by capital capacity prior to the achievement of full employment with consequent inflationary tendencies and balance-of-payments difficulties. This could take place even if Figure 2 were not characteristic of all sectors of industry.

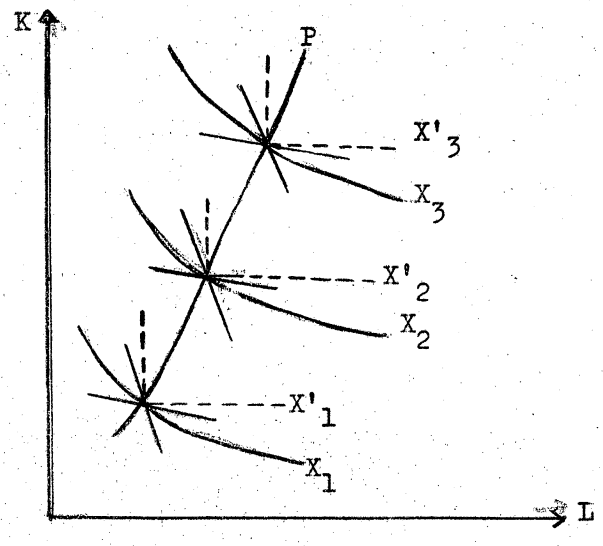

Figure 2

The Technological Restraints Hypothesis

It is fairly common for observers to report finding modern, capital-intensive equipment and techniques used in underdeveloped areas where relative factor prices would suggest the use of more labour-intensive techniques. I should now like to suggest that the use of the 'modern' techniques is not necessarily irrational emulation but the result of real limitations in the technological choices available, and that this, in turn, is a major source of labour-employment problems in underdeveloped areas. At this point the exposition will be oversimplified to indicate in stark outline the nature of the argument. In the next section the hypothesis will be combined with some of the market-imperfections hypotheses in an attempt to describe some of the major characteristics of underdeveloped and overpopulated areas by the use of a relatively simple theoretical framework.

The basic assumptions of the following analysis are: (1) in large sectors of an economy there are only a few alternative processes which can be utili-

zed; (2) these processes are relatively capital-intensive.[1] There have been frequent comments which describe certain features of underdeveloped and over-populated areas as essentially the result of limited variability in the coefficients of production. An example of this kind of comment is the frequently observed 'underemployment' in agriculture, where this is taken to mean that, with agricultural techniques remaining unchanged, withdrawal of farm labour would not reduce output.

The Case of One Good, Two Factors and One Process. In the first, most simple case to be considered, suppose that only one good is produced in the economy, national product which requires two factors, capital and labour.[2] Assume also that only one process can be used to produce national product, i.e., that the factors must be used in fixed proportions. This situation is prepresented in Figure 3, where the heavy black line represents national output, x_1 of 1 unit; the lighter lines represent higher outputs. Quite irrespective of relative factor prices, points a,b,c, etc., represent the combinations of factors which will be used to produce output and the slope of the line joining these points is equal to the constant, capital-labour ratio.

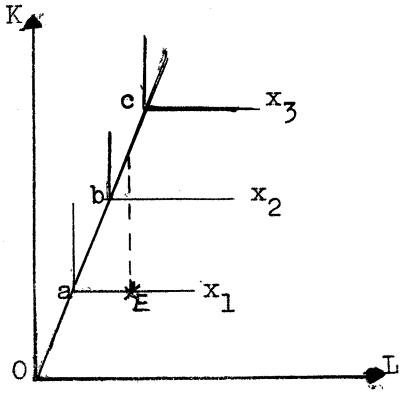

Figure 3

Only when the factors of production are actually available in proportions equal to the fixed capital-labour ratio is there the possibility that both can simultaneously be fully utilized. If the actual factor endowment is off the line Oabc, for example, at point E, there must inevitably be some unemployment of labour which is not amenable to any fiscal or monetary policy for its alleviation. Labour is a redundant factor and only by increasing

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1/ A production 'process' is a way of combining different factors of production whose proportions are determined by technology, although the scale of production and thus the absolute quantities of the factors used may be freely variable.

2/ Confining the analysis to only two factors is not essential but highly convenient for geometrical demonstrations.

capital stock in the amount indicated by the length of the dashed line can the unemployment be eliminated.[1] Conventional compensatory fiscal policy would, in this case, only result in inflationary pressures. The persistent open and 'disguised' unemployment in underdeveloped countries may be at least partially of this kind.

Two or More Processes. Suppose now that a second and a third relatively more labour-intensive process is developed for the production of the same good, national output, so that three processes are now available. This is represented in Figure 4 by the existence of two more right-angled constant-product lines for a unit of output and two additional expansion paths, Ocd and Oef.

In addition to the alternative combinations of factors which may be used to produce a unit output represented by points a and c and e, the lines ac and ce also represent combinations of factors which will yield a unit of output. It is possible to be between a and c on line ac, for example by using the first process and the second process in different combinations. If the resources are taken away from the first process, output would fall. But if these resources are then used in the second process, output would rise. It can be shown rigorously[2] and may be appreciated intuitively that there is some withdrawal of factors from process 1 and subsequent use in process 2 which will restore output to the unit level. Correspondingly the line bd represents combinations of process I and process 2 and of the two factors capital and labour which are optimal for the production of the x_2 level of output.[3]

In this second case where several processes are available the proportions in which the two factors can be used are not confined to either the expan-

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1 / I recognize that it is stretching a definition considerably to call redundant factors 'unemployed'. However, since it is specifically the hypothesis of this paper that the labour called 'unemployed' or 'underemployed' in underdeveloped areas is redundant, I shall, with this warning, use the terms interchangeably.

2 / Cf. Dorfman, op.cit., pp. 39-41

3 / It can be seen by drawing a line from a to e that, for any output, any combination involving processes 1 and 3 would require more of at least one factor than a combination of processes 2 and 3.

sion path of the first process or the expansion path of the second process, or both, but may be any place within the area bounded by Oab and Oef. Thus the factor endowment E_1, while inevitably implying some unemployment of labour when only the first process was available, can now be fully utilized by using the first process on a smaller scale and swithching some of the capital to the second process. If, however, the factor endowment should be outside the area bounded by the two expansion paths, at E_2 or E_3, for example, structural unemployment of capital capacity or of labour would ensue in exactly the same manner as in the preceding case of one process, regardless of factor-price ratios or fiscal policy.

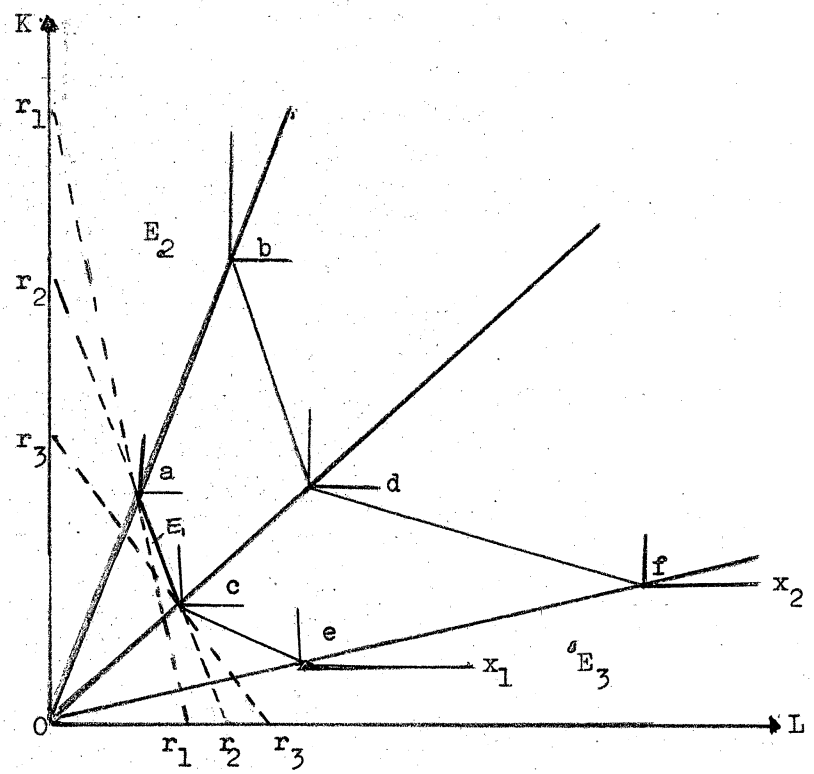

Figure 4

If more than two processes are available to the economy, full employment of all factors will be possible at a nonzero wage so long as the proportions in which factors are endowed fall on or within the limits set by the processes with the most extreme factor-use ratios. This suggest an observation which is, by now, almost trite: reduction of underemployment in overpopulated areas requires the addition of scarce factors. This, may, however, be accomplished in a variety of ways, such as the use of dry-farming methods and drought-and heat-resistant hybrids which increase the land available for farming.

In the case in which just one process is available, changes in relative factor prices cannot affect the proportions in which factors are used. This is not true when there are two or more processes available. In this case factor proportions employed will, in competitive markets, vary with factor prices and the achievement of full employment, if technologically possible with the given factor endowment, will depend on factor prices. In Figure 4 r_1, r_2, and r_3 are constant-expenditure lines illustrating three possible sets of factor prices. The line r_2 has the special feature that its slope is equal to that of the constant-product curve between points a and c.[1]

If the price ratio of factors were of the r_1 or r_3 types only one process and one ratio of factors would be used, To employ completely each of the factors in an endowment like E_1, the very special factor-price ratio r_2 would be necessary. A factor-price ratio slightly different from r_2 would be sufficient to move factor-use ratios away from E_1 to one of the isoquant vertices. Moreover, having reached a vertex of an isoquant it would be possible for large changes in relative factor prices to occur without leading to factor substitution.

Two Goods and Two Factors. An interesting question is whether the restriction of the analysis to only a single good is responsible for the character of the conclusion. By use of an Edgeworth-Bowley type box diagram we can continue to have the advantage of graphic techniques without loss of simpli-

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1 / Constant expenditures lines r_4 and r_5 could be drawn analogously to r_2, and r_1, with a slope equal to that of ce, and less than that of ce respectively.

city and extend the analysis to the case of two goods.

Let us now assume that we have two goods x_1 and x_2, each of which can be produced by two, fixed-proportions processes, and that constant returns to scale prevail,[1] only two factors, capital, K, and labour, L, are used. Figure 5a shows a few of the infinity of equal-product lines which could be drawn for different outputs of the two goods. The solid lines refer to product 1 , the dashed lines to product 2. In Figure 5b, these isoquants are use to contruct a box diagram. The dimensions of each side of the box represent the total amount of factors available. Any point within the box simultaneously represents four quantities: the amount of capital and the amount of labour used in producing x_1 which is determined by measurement from the lower left-hand corner, and the amount of capital and the amount of labour used in the x_2 industry, measured from upper right-hand corner.

Figure 5b provides the basis for the derivation of the 'efficiency locus' for the two goods x_1 and x_2. If production takes place at any point off this locus, it is possible by recombination of factors to produce more of one good without diminishing the output of the other. If along the efficiency locus corresponding amounts of the two goods are read off and plotted on a chart as in Figure 5c, the transformation or production-possibility curve between x_1 and x_2 is obtained.

To locate a point on the efficiency locus we must specify a

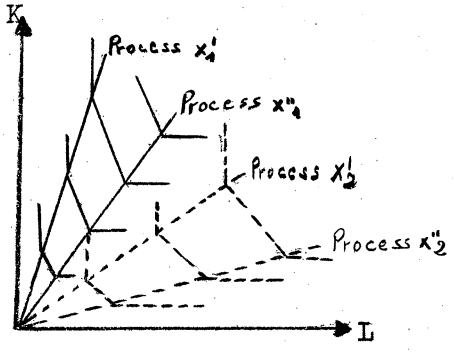


Figure 5a

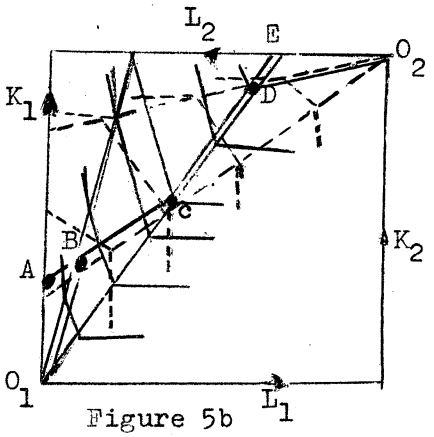
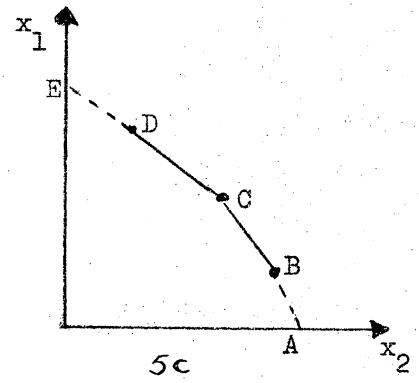


Figure 5b

1 / The assumption of constant returns to scale is , of course, maintained not because it is considered the best description of reality but for its analytical convenience. Some comments on the effects on the analysis of dropping this assumption are made below, pp, 367-8.



5c

particular amount of x_1 to be produced and find the maximum of x_2 which can simultaneously be produced. Graphically, we must move along the specified x_1 isoquant crossing x_2 equal product lines until we reach the highest x_2 isoquant obtainable. The optimum positions achieved will thus be located at tangencies of the x_1 and x_2 equal product lines where the lines just touch without crossing. Since, in the present case, each equal-product line is made up of segments of straight lines, the optimal positions will be corner tangencies. By repetition of this maximizing process for a series of points the entire efficiency locus can be determined.

Since in the present case the efficiency locus for the two goods is a rather complicated succession of line segments we shall trace it out carefully. Starting at O_1, zero output of x_1, the maximum output of x_2 obtainable is indicated at point A and could be computed by dividing O_2A by the scale factor applicable to process x'_2. If output of x_1 is now increased relative to x_2, it will be most efficient, at first, to use process x'_1 and for x_2 to be produced with process x'_2; production of x_1 would then move along the expansion path O_1B and x_2 should decrease along the expansion path x'_2 to point B. In this first stage capital is a redundant factor and labour is relatively scarce. The economic system adjusts to this condition by directing the use of the most capital-intensive processes to be used for the production of both goods.

As production of x_1 is expanded beyond point B it would be best now to use both process x'_1 and process x''_1 in combinations indicated by the intersection of O_2A with the constant-product lines for commodity x_1 as long as these intersections lie between the expansion paths of x'_1 and x''_1; production of x_2 should continue to be by means of process x'_2 alone, however. This second stage is indicated by the points on the segment BC which belong to both the combination of x'_1 and x''_1 and process x'_2. By tracing along an x_1 isoquant between B and C it can be seen that shifting to the expansion

path O_2A makes possible a larger production of x_2 for the particular output of x_1 than if we had remained on the path O_1B. As production of x_1 is expanded in this stage and production of x_2 is decreased, capital becomes more scarce relative to labour due to the relatively high labour-capital ratio of the resources released by the decrease in production of x_2.

In the third stage, as output of x_1 is farther increased, it is most efficient to use only process x''_1. But now for any given output of x_1, the maximum amount of x_2 can be obtained by use of the more labour-intensive process x''_2 in combination with process x'_2. The third stage on the efficiency locus is indicated by line CD.

Finally, still further expansion of production x_1 continues to be best done along the expansion path of process x''_1 until, at point E, x_1 is being produced to the complete exclusion of x_2. In this fourth and final stage the output of x_2 should be produced only by process x''_2.

Only in the fourth stage and the first stage of the efficiency locus would optimum allocation imply some unemployment of one of the factors. In the firs stage the unemployment of capital for different outputs of x_1 and x_2 is indicated by the vertical distance between lines AB and O_1B. In the final stage the unemployment of labour is measured by the horizontal distance between lines DE and DO_2.

Actually the occurrence and qualitative significance of any of the stages depends on both technology and factor endowments if process x''_1 were relatively more labour-intensive than is shown, its expansion path would pivot to the right and stage 2 in Figure 5b would be prolonged. As common sense would suggest, development of a sufficiently labour-intensive process for x_1 could cause stages three and four to disappear entirely and with them the possibility that there could be an 'optimal' configuration which involved unemploymemt of labour. A similar effect would result from a decrease in the amount of labour endowment. This could be depicted by squeezing together the left-and-right-hand sides of the box in Figure 5b. Increasing the labour supply would mean stretching the box horizontally. This would not only increase the range of outputs associated with stage 4 but also, if pushed far enough, first eliminate stage 1, the capital unemployment stage, and

then stage 2.

The points ABCDE on the technical transformation curve in Figure 5c correspond to the similarly lettered points on the efficiency locus in Figure 5b.[1] At first when only a little x_1 is produced and , relatively, a lot of x_2, we should move along the segment AB using process x'_1 and x'_2. Unemployment of capital associated with this segment on Figure 5b will be reduced as we approach B. Relative labour scarcity is limiting along this segment and the slope of the line segment AB will depend on the ratio of the labour inputs of output of x_2 to x_1. The relative labour intensity of process x'_2 compared to process x'_1 as drawn on Figure 5a accounts for the steepness of the segment.

The line segment ED on Figure 5c has an exactly analogous justification to that for the segment AB. Labour unemployment will be reduced as D is approached from point E. Capital is the only scarce factor and the relative capital intensity of process x''_1 as compared to process x''_2 accounts for the flatness of ED.

Point C is located conveniently relative to points B and D. More of x_1 is produced at C than at B, though not so much more as produced at point D. Likewise less x_2 is produced at C than at B though not as much less at D. The segments BC and CD will be straight lines as can be verified by noting in Figure 5b that, due to the assumption of constant returns to scale in all processes, there must be a constant ratio between changes in output of x_2 along the line O_2A between C and B, for example, and changes in output of x_1.

It was pointed out with regard to the efficiency locus in Figure 5b that changes in factor endowment and technology could shorten, extend or even completely eliminate various stages of the efficiency locus. This applies also to the separate segments of the technical transformation curve. The technical transformation curve of Figure 5c illustrates all the possible stages which could be produced by this simple case, from unemployment of ca-

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1/ In the constant-returns-to-scale- case, only relative factor endowments are important in determining the shape of the transformation curve. If the absolute factor endowments were changed while relative factor endowments remain constant, it would amount to sliding the north-east-and-south-west vertices of the box on the connecting diagonal.

pital to unemployment of labour. It should not be presumed that this range of possibilities will actually exist in a particular system at any one time. Rather, it is the hypothesis of this paper that technology and factor endowments in underdeveloped areas are such that a segment like DE, in which labour is redundant, is important in their transformation curves.

To demonstrate the importance of demand conditions for employment when the conditions assumed in the present hypothesis exist, we shall draw a transformation curve in Figure 6 consisting only of stage CD, along which there is full employment of both capital and labour, and the labour-redundant stage DE. This can be envisaged as the result of a high rate of population growth which has stretched the labour axis very far. We can now see that actual achievement of full employment depends on the relative demands for the two goods. Market baskets whose composition falls along CD will allow full employment; along DE labour will be redundant. A geometrical demonstration which is suggestive, though lacking in rigour, can be fiven. Suppose that the lines i_1 and i_2 represent two different possibilities for the community's indifference curve for the two goods. Only in the case in which the indifference curve is tangent along CD will optimal output imply full employment. The community must face a conflict in goals between full employment and maximum value of output if i_2 is in fact its indifference curve.[1] Extending the analysis to include many goods would

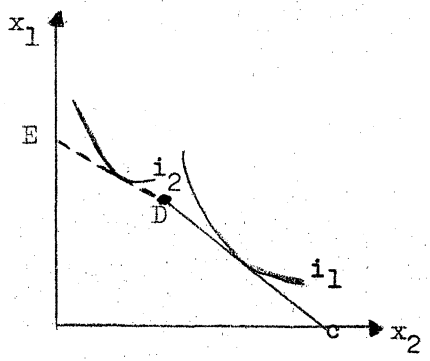

Figure 6

1/ A major qualification to this analysis, still on the static level, is the possibility that the shape and position of the community indifference curves might not be independent of the particular processes or combinations of processes which are used, To handle this difficulty it would be necessary to determine the shifts in income distribution which result from changes in factor prices and to explore the differentials in tastes of the recipients of the different types on income.

widen the range within which factor endowments could vary without unemployment of one or more factors resulting. There would, however, still be no guarantee that the composition of goods demanded would always hit on a full-employment point.

It may also be observed that if it is possible to buy and sell in foreign trade at price ratios between the slopes of CD and DE, full employment would again be possible though not necessary. To determine whether or not it would result, it would be necessary to know the reciprocal demands for exports and imports.

It would be possible to elaborate this model now by investigating the implications for the analysis of market imperfections such as were considered in the previous section. This extension will be postponed to the following section, however, and applied to a model of underdeveloped areas designed to be somewhat more realistic. As it stands the present analysis gives us, I believe, important insight into the problems of underdeveloped areas. It presents a hypothesis which helps account for the inflationary tendencies of underdeveloped areas under the impact of programmes designed to raise effective demand, and for the stubbornness of unemployment in such areas.

The analysis of this section also provides a more precise definition for the 'technological unemployment' mentioned in Measures for the Economic Development of Underdeveloped Countries.[1] Technological unemployment may be a real problem for underdeveloped areas if it is defined, as in this section, as redundant labour arising from resource and technological restraints and the structure of demand.

11. A MODEL OF UNDERDEVELOPED AND OVERPOPULATED AREAS[2]

Though the analysis of the previous section is suggestive, it is hard to believe that all of the unemployment and under-employment in underdeveloped areas represents literally useless labour. Moreover, the assumption of

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1/ United Nations, Department of Economic Affairs (New York, 1951).

2/ I am particularly indebted to P.N. Rosenstein-Rodan for discussion of the issues raised in this section.

only a few alternative processes and a quite limited range for substitution of factors does not seem to fit well the technological characteristics of a number of important industries, as, for example, agriculture. I shall attempt therefore to move towards greater realism by use of a two sector model (one section with fixed, and one with variable coefficients of production) and investigation of the effects of market imperfections in such a system. To the assumption of limited opportunities for substitution in some industries is added the hypothesis that in many other industries there is a considerable range of variability in the proportions in which factors can be used.

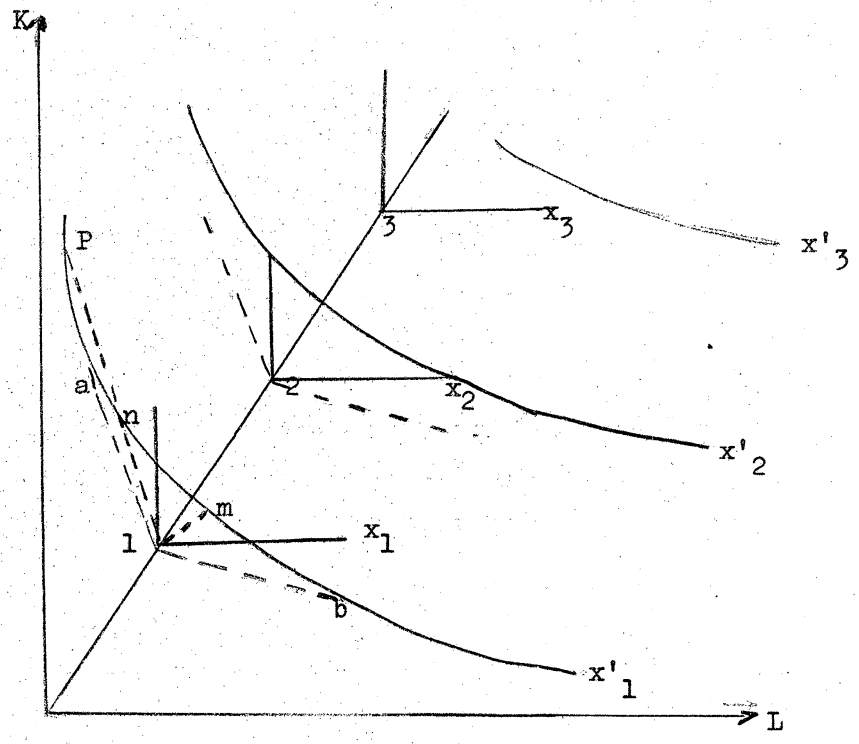

Figure 7

It will be useful to initiate the discussion under the assumption that each of the two sectors produce the same product. Suppose that in Figure 7 the constant-product lines of the fixed-coefficient industry are represented by

the lines x_1, x_2....[1] and the constant-product lines of the variable coefficient industry by the lines x'_1, x'_2....

The output x_1 could be produced by the factor combination and technique represented by point 1 or any of the factor combinations using the variable-proportions technique represented by the line x'_1. Moreover, following the reasoning on page 356 above, it is also possible to produce x_1 by simultaneously using both the fixed-coefficients and variable-coefficients techniques. All of the lines which could be drawn from point 1 to line x' represent a combination of methods which would produce output X_1; all such lines fall between the lines 1a and 1b which are drawn from point 1 just tangent to line x'_1.[2] The 'efficiency locus' for specified outputs can be traced out be determining, for given amounts of one of the factors, the minimum amount of the other factor necessary to produce the output. If this is done for output x_1 when very little labour is available it is best to produce by the use of the variable-coefficients process alone; a representative factor use for this case is at point p. As more labour becomes available the minimum amount of capital required to produce x_1 is found by sliding down the variable-coefficients constant-product line to point a. Line a1 represents different combinations of the variable-coefficients technique located at a and the fixed-coefficient process. When the labour available is further increased, the minimum amounts of capital necessary to produce x_1 are found by moving alone line a1. As labour available is still further increased the line 1b is the next segment of the efficiency locus used, for reasons analogous to those given for the use of segment a1. Finally, when labour is increased beyond the amount available at point b, again only the variable-coefficients method should be used to produce x_1.

Output x_1 could also be produced by process combinations and amounts of factors which do not lie on the efficiency locus, of course. Line p1 repres-

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1/ Although the constant-product lines for the fixed-coefficients sector are drawn in Figure 7 as if only one process is available, the demonstration is perfectly general and its implications are applicable when more than one process is available for the fixed-coefficient industry.

2/ I am indebted to R. Solow for the criticism of a previous paper which led to this formulation.

ents a series of such combinations, using in varying proportions the variable-coefficients techniques located at p or n and the fixed-coefficient process located at point 1. Any combination of methods along pnl, however, would result in higher costs for x_1 than a method found on the efficiency locus; methods along pnl could also be used to produce larger amounts than x_1. Of course, many lines like pnl could be drawn between a 1 and the vertical portion of the fixed-coefficient x_1 isoproduct line and, analogously, between 1b and the horizontal portion of the fixed-coefficient isoproduct line. Lines like 1m, of which many could also be drawn, represent combination of methods which would also produce x_1, but require more of both labour and capital than points on the efficiency locus. The boundaries of lines such as 1m are the vertical and horizontal portions of the fixed-coefficients x_1 isoproduct line.

Figure 7 embodies the constant-returns-to-scale assumption for both the fixed-coefficients and variable-coefficients method. This is not necessarily the most realistic or relevant assumption however, nor does the relative position of the two types of curves, or the shape of the variable-coefficients isoquants necessarily correspond closely to reality. It is useful to recognize other, special cases which may have important empirical significance. In Figure 7, for example, only the extremes of the isoproduct curves of the variable-coefficients technique were a part of the efficiency locus for any particular output, and, as drawn, relatively little substitution was possible at such extremes. It would, of course, be possible to draw figures in which the 'efficient' isoproduct ridge lines follow the variable-coefficients lines so as to allow substitution of factors over a considerable range.

The effect of divergent rates of return to scale on the shape and slope of the constant-product ridge lines is illustrated in Figure 8 for one possible set of relations. In the fixed-coefficients process it is assumed that there are increasing returns to scale (shown by decreasing distances between x_1, x_2, x_3, along any ray from the origin). In the variable-coefficients method constant returns to scale are the rule (shown by constant distances between x'_1, x'_2, x'_3, along a ray from the origin). In this example, the efficient isoproduct lines change their shape as output is increased. For output x_1 only the variable-coefficients constant-product line is relevant. For output x_2 the 'efficient' isoproduct ridge line involves use of

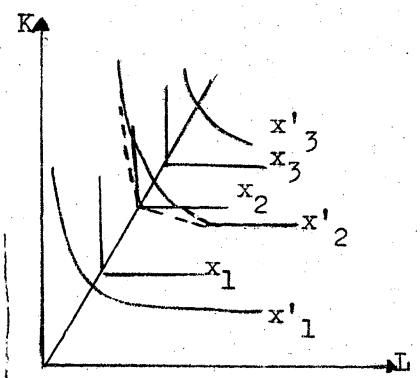

Figure 8

both the fixed-and variable-coefficient techniques. Finally, for output x_3, only the fixed-coefficent technique is 'efficient'and the ridge lines involving the variable-coefficient method necessarily have a positive slope.

Figure 8 provides the formal basis for some useful deductions. Changes in factor prices which might at one scale of output induce shifts in the proportions in which factors are used, may not induce such shifts at another scale of output or may only produce smaller shifts. Likewise techniques of production not feasible at one scale of production may become mandatory for efficiency at another scale.

In Figures 6,7 and 8 it is particularly clear that in order for the system to travel along its most efficient production isoquant it is necessary that factor prices be flexible. Factor-price rigidities would make parts of the efficient constant-product lines unattainable for profit-maximizing businessmen.

We can now go rapidly through an analysis of the two-sector model hypothesis assuming that each sector produces a different commodity. The geometrical representation of this, using the box diagram technique introduced in the previous section is provided by Figure 9a. The assumption that for x_1 only two alternative processes are available is maintained for convenience; the resulting production-possibility curves for x_1 are shown by the solid lines. The assumption of variable coefficients in the production of x_2 is limited to the sector between its ridge lines because at these ridge lines the marginal productivity of one of the factors becomes zero and further input of this factor would have no effect on output.[1]

The production functions for x_1 and x_2 are reproduced in the box diagram

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1/ It is assumed that there is no disposal problem and, thus, that the production isoquants do not bend back on themselves.

in Figure 9b: the dimensions of the box are determined by the factor endowments .[1] Using this box diagram we can trace out the efficiency locus for the two products by repeatingly asking the question, 'For a given amount of x_1 what is the maximum amount of x_2 which can be produced?' In the process of tracing out the efficiency locus, the transformation curve can be drawn for the two goods.

Starting at O_1, zero output of x_1, the maximum amount of x_2 producible is given by O_2A. If the output of x_1 is increased relative to x_2, it would be most efficient at first to use process x'_1 for x_1 and to produce x_2 by travelling along its most capital-intensive ridge line. O_2A. This represents optimal behaviour up to point B, In this stage although both products are being produced capital is a redundant factor. This results in spite of the variability of coefficients in production of x_2 because outside the upper ridge line of x_2 capital has zero marginal productivity. Stage 1 is represented on the transformation curve in Figure 9c as segment AB.

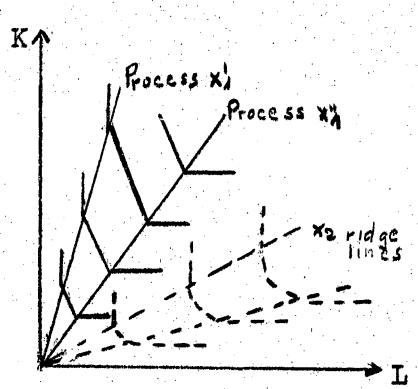

Figure 9a

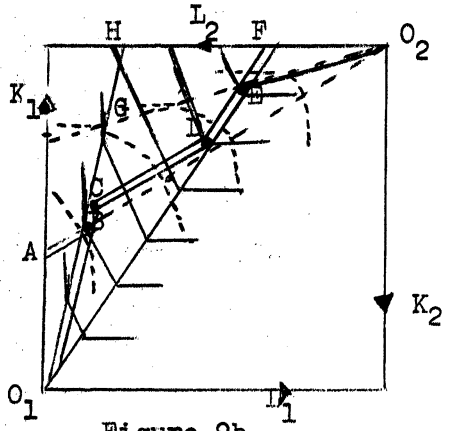

Figure 9b

The efficiency locus from B to C is traced out by finding the succession of points at which the x_1 constant-product curves touch the highest x_2 constant-product curves. In this stage process x'_1 will be used for x_1 and a varying combination of factors for x_2. The segment of the transformation curve

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1 / Again only the ratio of the factor endowments is important as long as constant returns to scale is assumed.

for this stage, BC, will be curved as equal increases in production of x_1 along x'_1 will not result in constant changes in output of x_2. Only along rays from the origin O_2 will equal distances imply equal differences in output of x_2. BC will be concave to the origin as is 'normal' for transformation curves; graphically it can be seen that smaller and smaller changes along x'_1 are needed in order to move across equal changes in output of x_2. As production moves from B to C the points at which x'_1 corners touch the x_2 isoquants will be characterized by smaller and smaller slopes on the x_2 isoquants, corresponding to the decreasing capital-labour ratio used in the production of x_2. At some point, C, the capital-labour ratio in x_2 will become equal to the capital-labour ratio represented by the negatively slanting portion of the x_1 isoquants. A ray from the origin O_2 to point C will intersect every x_2 isoquant at a point with identical slopes. Thus CD, the third stage of the efficiency locus and the transformation curve will be the series of tangencies of the negatively, slanting portion of the x_1 isoquants, representing combination of processes x'_1 and x''_1 and the x_2 isoquants along the ray O_2C. The segment of the transformation curve corresponding to CD lies on a ray from O_2 and thus the equal jumps across the x_1 isoquants will mark out constant changes in production of x_2.

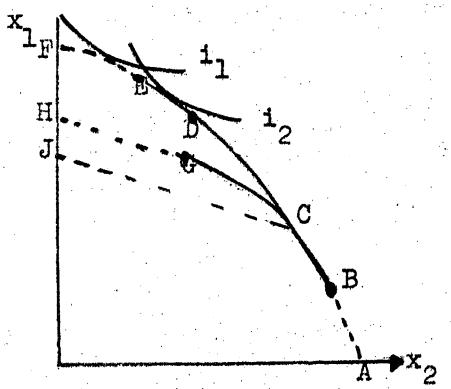

Figure 9c

At point D and for further increases in output of x_1 relative to x_2 it would be best to use only process x''_1 for production of x_1. The segment of the transformation curve, DE, corresponding to DE on the efficiency locus, is curved for reasons similar to those which created the curvature of segment BC.

The final stage of the efficiency locus is the labour unemployment stage. The marginal productivity of labour has fallen to zero in the x_2 sector and in a perfect factor market wages would fall to zero. EF represents this final

stage on the transformation curve in Figure 9c.

ABCDEF in Figure 9c is the full transformation curve for this case. There are now curved as well as straight-line-segments, and the kinks characteristic of the previous transformation curves have disappeared. [1]

In Figure 9b, as in Figure 5b, it is possible to visualize the effects of structural changes by altering the shape and position of the production isoquants for each product and the dimensions of the box.

The box diagram approach helps to clarify the implication of the differences in substitutability in the fixed-coefficient and variable-coefficient sectors. It is clear that the outputs at which one or another factor becomes redundant will be determined by the limits of subtitution in the variable-coefficients sector and the most extreme labour-and-capital-intensive processes in the fixed-coefficient sector, not by the discontinuities of the latter sector. These have other important effects, however.

Suppose that the respective demands for output are such that a large part of the available capital is drawn into the capital-intensive and fixed-coefficient sector. The amount of labour which can be absorbed in these sectors is dependent on the amount of capital available. Since capital is a scarce factor, labour employment opportunities in this sector are limited by its availability rather than by demand for output. The relatively plentiful labour supply is then pushed into the variable-coefficient sector and absorbed there as long as the marginal value productivity of labour is higher than the wages it receives.

In this case, as in the models of the preceding sections, unemployment is not due to lack of effective demand and as a result cannot be relieved by conventional contracyclical economic policy designed to stimulate spending. If employment opportunities in the fixed-coefficient sectors were limited by capital scarcity or some other resource bottleneck, an increase in demand rather than stimulating additional output would only create inflationary pressures. Likewise in the variable-coefficients sectors if the marginal product-

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1/ Being off the efficiency curve, it may be noted, is like being on an isoquant 1m in the single-good case. Cf, Figure 7 above.

ivity of labour were zero, the first effects of an increase in demand for output would be an increase in prices without an increase in production. If more of the scarce factors were made available to the fixed-coefficient sectors, more labour could then be employed and would be used, if there were an effective demand for its output. Additional amounts of the scarce factors in the variable-coefficients sector would also increase labour productivity and output if demand were adequate.

It is possible in this case of two sectors, one of fixed and one of variable coefficients, for a divergence to exist between the full-employment output and the output with maximum value, just as in the case of fixed-coefficient processes for each of two goods, as depicted in Figure 6 above. This could result even if there were no market imperfections. If, in Figure 9c, the community indifference curves were like i_1 rather than i_2 so that the tangency occurred in the capital-scarcity, labour-surplus stage, the divergence could exist. On the other hand, community-indifference curves shaped like i_2 would mean that it would be possible for full-employment output and maximum-value output to be indentical. This demonstration is subject to the same qualifications applied to the one-good case.

Without empirical knowledge, it is not possible to evaluate certainly the relative importance of each of the stages of the transformation curve 9c. However, according to the hypothesis advanced here the transformation curve for underdeveloped areas would consist mainly of the high labour-intensity and labour-unemployment segments such as DE and EF.

We have thus far in this section assumed the existence of competitive markets and profit-maximizing entrepreneurs. We have shown that , even under such assumptions, technology, factor endowments and final demands may combine in ways which make it very difficult for underdeveloped areas to solve their problems of unemployment and underemployment. It is possible now to broaden the analysis by combining it with the analysis of the effects of market imperfections discussed in Section 1 to determine what further problems are created when some of the assumptions of competition are dropped.

Behind the transformation curve lie many fine adjustments as factors are shifted from one industry to another and recombined in varying proportions

to obtain the maximum output from one industry for given outputs of the other industry. It has been assumed in deriving the transformation curve that the necessary adjustment would be accomplished as they would be in a perfect factor market. But flexibility within wide limits is required to achieve every possible position on a transformation locus such as that in Figure 9c. When imperfections and rigidities of various types obstruct the movements of factors and prices, the system will not be able to achieve its optimum transformation curve but will instead do no better than to move along some other, less than optimal curve. Limited factor-price flexibility may be quite serious when at least one good is produced with fixed-coefficient processes. If rigid factor prices render a relatively labour-intensive process unprofitable, the only alternative process may involve a big jump to a quite capital-intensive process as weel as a drastic obstacle to substitution in the variable-coefficients process.

Since we are interested in the effect of imperfections in the factor markets, it will be useful to distinguish transformation curves which assume perfect adjustments in the factor markets; these will be called 'technical transformation curves'. Transformation curves which take into account market imperfections will be different from the technical transformation curves; these latter relations will be designated 'market transformation curves'.[1] Different types of market imperfections will create different types of shifts away from the technical transformation curve, so there is not one market transformation curve for each technical transformation curve but many.

It may help to approximate reality to assume that factor-price ratios in the variable-coefficients sector are relatively more flexible than those in the fixed-coefficients sector (are relatively more flexible than those in the fixed-coefficients sector). This might result from differential strength

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1/ The effect of factor-market imperfections in shifting the market transformation curve inside the technical transformation curve has been pointed out and analysed for international trade by G. Haberler, 'Some Problems in the Pure Theory of International Trade', The Economic Journal, June 1950, LX, 223-40 and by P. Samuelson, 'Evaluation of Real National Income', Oxford Economic Papers, Jan. 1950, 11 (N.S.), 18-19 for welfare economics; others have probably also noted the effect.

of union organization or susceptibility to government wage controls. Suppose for example, that by means of union pressures or minimum wage legislation real wages of labour were maintained so that in the fixed-coefficients sector the labour-capital price ratio was set above that represented by the slope of the constant product curve combining processes x'_1 a x''_1 in Figure 9b; the factor-price ratio in the variable-coefficients sector may still be assumed to vary freely. The cost-minimizing combination of factors is determined for any particular output at the point at which the production-possibility schedule for that output touches the lowest expenditure line. If the production-possibility schedule has any slope at this point, it is a condition of equilibrium that this slope be equal to the factor-price ratio which determines the slope of the expenditure line. As a result under the present assumptions process x''_1 would never be used, It would always be more profitable to use process x'_1 alone in producing x_1. The transformation curve would be ABCGH in Figure 9c; this is below the technical transformation curve and has a much longer range of unemployment. If both sectors were characterized by such high, inflexible factor-price ratios, the economy's transformation curve would approach ABCJ.[1]

Barriers to the movement of factors would have the effect of moving the market transformation curve even further inside the technical transformation curve and increasing the range over which a factor is redundant.

Thus inperfections in factor markets have several undesirable effects. They reduce the amount of goods available and create a wider range of combinations of goods over which labour may become unemployed, depending on the structure of final demand.

Although the effects of rigid wages on the transformation curve of the economy are clear, welfare judgements as to the results of removal of the factor-price rigidities are subject to the same qualifications as in the previous case. Much depends on the effects of a change in methods on the income distribution, and, via income distribution, on community preferences.

1/ Barriers to capital movement created by monopoly may create situations analogous to those described above resulting from labour-market imperfections.

III. EMPIRICAL VERIFICATION

The analysis above is based on hypotheses which can be tested empirically and which deserve to be given factual content. Empirical testing requires measurement of the proportions in which productive factors can be and are actually used. Essentially the objective of such research woulb be an investigation of production functions.

Although the concept of the production function has been familiar for some time, its empirical investigation has, as is common, been neglected with important noteworthy exceptions. In this conexion it should be noted that even though it is sometimes presented as an analysis of productive processes and its terminology is taken from the theory of production, input-output analysis, as it now stands, does not reveal the technologies actually in use in an economy. For this purpose a much higher degree of disaggregation would be necessary than is currently practiced or appears feasible. Interindustry flows may strongly reflect historical incidents by which certain technical processes are concentrated in a particular sector which subcontracts for other sectors. Or, if similar technical processes are widely used, the interindustry flows in a particular year may reflect different cyclical patterns in industry; one industry producing at its capacity may subcontract to another industry which has equipment capable of performing the necessary operations and is cyclically depressed.[1] Studies of capital coefficients made for input-output tables do indicate, however, a method which can be used for a factor-proportions study.[2]

The objective of the process analysis approach newly developed at the Rand Corporation is exactly the empirical determination of production function. The emphasis of the process analysis approach in establishing all the alternatives on a production-possibility schedule, which is essential for programming, is somewhat different from a factor - proportions study. For the

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1/ This point is made with force by H, Markowitz in Process Analysis of the Metal Working Industries, The Rand Corporation (Santa Monica, 1953), pp. 7-8.

2/ Especially the studies made by the Inter-Industry Analysis Branch of the Office of Chief Economist, Bureau of Mines, U.S. Department of Interior.

latter study not only the range of possibilities but the relative frequencies of their use and dynamic considerations involved in choice are important. The methods of process analysis can also be used for a factor-proportions study, however.

For the study of factor proportions two general approaches seem to be available. The first, suggested by input-output studies of capital coefficients and which will be called the 'product analysis' method, involves a census in each plant studied of the amounts of each type of factor of production used in the expansion of the output of a particular product. The second method, based on classifications of technical processes, requires the determination of the combinations of factors actually used by firms to perform certain standardized 'tasks'. These approaches to the factor-proportions study are not necessarily logically separate, nor should they always be completely different in application, The great advantage of process analysis in precise identification of outputs can compensate for the weakness of product analysis where multi-product plants are involved. The advantage of product analysis in inclusion of all contributing inputs can be important when using the process-analysis approach where it is difficult to isolate the contributions of all inputs. Thus, it is important in particular cases to have in mind a method combining both theproduct and process analyses.

The application of product analysis can be made in two ways: (1) By abstracting the data required from the engineering plans which are prepared when a new investment is undertaken and which list the construction, equipment, labour and materials required for the operation of the plant. Accounting records of new investment expenses could also supply part of the necessary information. (2) By means of 'factor inventories' of existing plants to provide for these plants the information which the investment plan analysis provides for new expansions. No easy and automatic application of product analysis techniques is possible in the face of problems such as those raised by multiproduct firms and the measurement of the expansion of capacity. These problems can often be overcome, I believe, to make this a fruitful method.

Process analysis is based on the conception that all productive activity

can be divided into separate technical processes with similar outputs whose inputs can be identified and compared. Process analysis thus provides another logically satisfactory approach to study of factor-proportions. The process-analysis approach can moreover provide the basis for a comparison of factor-proportions by final products, and thus for an independent check of factor-proportions computed by the product-analysis method. This could be achieved by determining the appropriate physical processes and levels of activity necessary for the output of a particular final product and aggregating their factor inputs. The procedure just described is, in fact, that actually used in modern engineering practice in plant and equipment design and layout.

The disadvantage of the process-analysis approach, however, stemming from the kinds of information which would be generally available. I believe occurs precisely where the product-analysis method is strong. The information for the process-analysis approach must come from the records of inputs to particular processes and these records, because of the purposes for which they are kept, will seldom be sufficiently detailed and conprehensive as to the inputs involved in a process. As a result it will often be necessary in using the process-analysis approach to estimate the contributions of 'indirect' inputs to the processes studied.

A major source of information for the application of the process-analysis approach may be the time cards kept by many firms, These cards list for each worker the time which he takes at each type of machine which he uses to perform the operations on the particular piece. These cards would have the labour inputs and machines specified for particular tasks and often contain other useful data as well. The job sheets which accompany production orders are also sources of information as they list the time per unit and in total required by each type of machine and process to finish a particular item.

CONCLUSIONS

In this paper a number of different hypotheses have been developed and combined for the purpose of explaining outstanding features of some underdeveloped areas: the persistence of unemployment and underemployment, the coexistence of 'modern'capital-intensive techniques and methods using a great deal of labour and little capital, and large differentials in factor

returns in different sectors. I have suggested that to a considerable degree these conditions may be the result of a few characteristic conditions: factor-market imperfections, and limited technical substitutability of factors, with divergences between the proportions in which goods are demanded and in which they can be supplied with full use of available factors.

Factor-market imperfections which limit factor mobility create employment problems in underemployed areas with low per capita incomes and limited capital resources which are not different in kind but are much different in degree from those existing in the more advanced countries.

When the proportions in which factors can be combined are variable without limit, i.e., with decreasing but always positive marginal returns to labour, additional labour can always produce additional output. If the technical substitutability of factors is limited, as is suggested here, the possibility of labour redundancy arises. Even if there are some sectors in which labour always has a positive marginal product there may be a divergence between maximum value output and full employment output if there is insufficient demand for the output of these sectors. These possibilities, are again more important for the underdeveloped areas whose resource endowment are often not suited to the factor proportions dictated by the technological leadership of advance countries. Differences in income distribution and the range of products may also make limited technical substitutability a more pressing problem in underdeveloped than in advanced areas.

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