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MATHEMATICAL MODELS IN HEALTH SERVICES PLANNING

by

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I. INTRODUCTION

A system is defined as a "regularly interacting or interdependent group of items forming a unified whole ... as ...: a group of interacting bodies under the influence of related forces ...".[1] A personal health services system consists of interdependent elements, such as physicians, nurses, facilities and other resources, that interact under the influence of diverse forces, with the community they serve.

The elements of any health services system can be grouped in subsystems that depend on the criteria for the grouping, e.g., first contact care or primary medical care, specialist care or consultant medical care, community hospital care, teaching hospital care, etc. The term subsystem is interchangeable with the term level or state of care.

The state of a health services system can be defined by the values of those variables that describe its elements, e.g., prevalence of a particular disease, available hospital beds, etc. as well as by the process of transformation in the system whereby inputs are translated into outputs.

The input into each state can be measured by the number of entries i.e. persons or conditions, as determined by the actual "demand" for services per unit of time, e.g., a patient who twice visits a consultant specialist during a year because of otitis media constitutes an entry to the consultant care state with two visits for that entry into that state during the year. If need is preferred to actual demand, the input in the model can be changed to a desired potential demand. Such a shift assumes that need, i.e. the submerged part of the iceberg of disease, can be translated into demand.[2] The conceptual distinction between these two approaches has been discussed somewhere else.[3] The parameters that define this input will depend on the criterion chosen to define such measures of ill health as disease, disability, dissatisfaction and discomfort[4].

The output of each state can be measured by the number of discharges or outcomes from each per unit of time. Alternative outcomes might include: dead/alive, diseased/healthy, disable/fit, dissatisfied/satisfied, or uncomfortable/comfortable.

The throughput represents the time movement of patients through the several states of the system. This movement within the system can be between two units within the same state of care, i.e. a transfer, or between units of different states, i.e. a referral.

Transfers and referrals document the movement or flow of people within a health services system and the dynamic relationships among its different states and units. The series of referrals and transfers experienced by each patient defines his utilization experience and reflects the utilization strategy employed[5]. Thus, the throughput of the entire system can be defined as the totality of utilization experiences for all patients.

INSERT FIGURE I.

Figure I shows two examples of utilization strategies.

II. MODELS BASED ON THE KNOWLEDGE OF THE PERFORMANCE OF THE SYSTEM

Planning personal health services can be based upon analysis of the performance or the structure of the system. In the methods based upon the performance of the system, the resources required are determined by the amount and type needed to achieve a certain output, called product output, which is measured in terms of performance, such as reduction or control of death, disease, disability, discomfort, etc. In those methods based on the structure of the system, the output is defined as process output and measured in terms of services provided or population covered. Effectiveness is the relationship between input and output in the system performance method[6]; efficiency is this relationship in the system structure method.

Unfortunately, little is known about the effectiveness of different health services. Most analytical studies of health services have been concerned with productivity, expressed in terms of efficiency, but not with effectiveness. The paucity of effectiveness studies is due to present limitations in the knowledge of the relationships between the different variables involved in the output as well as in the input of the system and their interrelationships. In most cases the relationships between the system and its performance are not known; even less is known about methods of quantifying them. There is no evidence, for example, that in providing X units of prenatal care one will save Y children's lives. It is in the study of these relationships that epidemiological studies are greatly needed. Only on those cases with a known quantifiable relationship between input and product output, such as kidney dialysis and prevention of death in certain renal failures, is it possible to use techniques, such as cost-benefit or cost effectiveness analysis, that require that knowledge[7]. Otherwise, the usefulness of the technique is conditional on the validity of the assumptions on this relationship.

The absence of objective standards to measure the relationships between systems and their product output explains the use of subjective measures, such as the opinion of experts. The Centro de Estudios de Desarrollo (CENDES) and the Pan American Health Organization[8] and the U. S. Public Health Service[9] have defined planning methods that require these experts' judgment about the vulnerability of the disease to certain curative and preventive activities.

III. MODELS BASED ON THE KNOWLEDGE OF THE STRUCTURE OF THE SYSTEM: MARKOVIAN MODELS

In methods based upon the knowledge of the structure of the system it is convenient to use probabilistic models. These allow more flexibility to planners in facing a continuously changing and uncertain environment. Indeed, biological and social models are mostly probabilistic in nature[10].

With the work by Markov (1856-1922) an important step in the development of probability theory was taken. He initiated a basic study of sequence of events with a given distribution of initial probabilities, which have the simple property that the probability of the next event in the successive sequence of trials depends only on the present outcome rather than on the particular occurrence of any previous one[11]. These situations, now called "Markov chains", have been broadly studied, and modern expositions of Markov chains are contained in the books of Doob[12].

Feller[13], Kemeny and Snell[14], and Bartholomew[15].

Since the first application of Markov chains in stastical mechanics, there have been many more applications in surprisingly diverse areas. These include the work on learning theory independently developed by Estes[16] and Bush and Mosteller[17]; the study of changes in attitudes by Anderson[18]; the analysis of social mobility by Prais[19] and labor mobility by Blumen, Kogran, and McCarthy[20]. In epidemiology, Marshall and Goldhammer[21] have used Markov chains for the study of epidemiology of mental disease, and Fix and Neyman[22] and Zahl[23] have used it for the study of survival after treatment of cancer. In planning personal health services, Navarro and Parker[24], Singer[25], and Hope[26] have advocated the use of the Markovian chain as a mathematical model to estimate manpower and facility requirements.

A MARKOVIAN PLANNING MODEL

This model embodies a Markov chain, in which the health services states are postulated and the probabilities of going from one state to another, defined by the transitional probability matrix, determine the number of people in the various states throughout time. In other words, the transitional probability of going from one health services state to another depends only on the current state that the patient is in, not on any previous states that have led to his current state. In addition, the transitional probabilities are assumed not to vary with time[27].

In the present application the assumption is made that every person in the population of a defined geographical region is characterized as belonging to one, and only one, of several mutually exclusive states of a health services system at any point in time.

INSERT FIGURE II.

The health services states shown in Figure II have been chosen arbitrarily. The state described as "Population not utilizing medical or hospital care" includes all persons who are not in any other state. It includes healthy as well as sick persons who are not under medical care in any of the other states. Primary medical care, consultant medical care, community hospital care and teaching hospital care states contain people receiving these levels of care respectively. The number of states could be extended by adding other states of care as well as different units within each state. The size of the model can be extended in accord with the complexity and comprehensiveness desired and the availability of usable information. The population to be examined can be defined by demographic and/or epidemiological criteria[28].

In Figure II to say that n_2 equals 20 persons means that at this moment, $t = 0$, there are 20 persons under primary medical care.

The fractions of the population at each state, at different time periods, $P_i(t)$, equals the number of people in that health services state at that time, $n_i(t)$, divided by the total number of people in the region served by the system at that time, $N(t)$.

For a stationary Markovian chain analysis, the transitional probabilities of going from one state to another during the fundamental time period must be calculated. The transitional probability, P_{ij}, equals the number of people, n_{ij}, who are transferred from state i to state j during the defined time period, divided by the number of people, n_i, in state i at the beginning of that period.

This transitional probability denotes the probability that a person being in state i at the beginning of the defined time period will go to state j during that period. For example, if n_4, the number of people under community hospital care at a beginning point in time is 200, and n_{45}, the number of referrals from community hospital care to teaching hospital care measured in the week that follows is 50, then $P_{45} = \frac{50}{200} = 0.25$ is the measured transitional probability per week of going from community hospital to teaching hospital. P_{ij} defines the movement of people within the system and reflects functional relationship among the states.

INSERT TABLE 1.

Table 1 presents the transitional probabilities in the described model. Each transitional probability represents a flow between two different health services states. P_{24} for instance, represents the probability that a person is in the primary medical care state at the beginning of the chosen time period and goes to the state of community hospital care during the period.

In this model P_{ij} is taken as known. It is determined from information about referrals within the system (Appendix 1). It would be possible, however, for those populations where such data are available, to relate

P_{ij} as the dependent variable in a multiple regression analysis, considering as independent variables those variables which condition utilization from the standpoint of the persons, of the system, and of enabling factors.

If the fractions of the population in different health services states are known, and if the parameters that define productivity are known, the manpower and facilities required can be calculated (Appendix 2).

APPLICATIONS OF THE MODEL

PREDICTION

Prediction is the ordinary statistical problem of forecasting. At the simplest level it involves extrapolation of past experiences into the future.

In the Markovian model when the transitional probabilities are known, prediction is possible when only the initial fractions of the population in each state are known (Appendix 3). Prediction involves calculating the fractions of the population expected to be in the several health services states at different time periods in the future. The inputs for this model of prediction are the known current fractions in each state and the transitional probability matrix that reflects the dynamics of the system. The outputs of the model are the estimated fractions of the population in each state at different time periods in the future. If the productivity of current resources is known it is possible to estimate the manpower and facilities required in each state in the future.

SIMULATION

Simulation involves observation of changes in the health services system and the repercussions of those changes on present and future utilization and resources.[29] The inputs of the model applied to simulation are the fractions of the population currently in each state and the new set of transitional probabilities that reflects simulated changes in the system. The outputs are the new patterns of utilization determined by the changes. Since the productivity of the resources is known, these new fractions in each state can be translated into a new set of resources.

GOAL-SEEKING

Goal-seeking involves determining that alternative which minimizes "costs" or "changes" in resources, required to achieve, in a given time period, specified utilization patterns or specified needs for resources.

The inputs of the Markovian model in goal-seeking are: the present fractions of the population in each state, the desired future steady state fraction in each state (or the desired number of resources in a particular state), and the selected constraint, e.g., a cost constraint, a resource constraint, etc., that the alternative specified must meet. The problem is to choose that alternative, defined by a transitional probability matrix, which will minimize the constraint selected (Appendix 3). Actually there will be an infinite number of possible alternatives in going from the present level of utilization to one desired in the future, but only one alternative will minimize the constraint selected. For instance, if the constraint is "cost", then the alternative chosen will be the one that minimizes the cost of going from the present to the desired future level of utilization. Another example of constraint might be "minimizing changes" and, in that case, the alternative chosen would be the one which would require minimal additional resources for each health services state at different future time periods.

INSERT FIGURE III

Figure III illustrates the three applications of the Markovian model.

THE MARKOVIAN MODEL APPLIED TO PLANNING FOR CARDIOVASCULAR DISEASES

This example deals with the application of the model described above to the planning of personal health services for patients with cardiovascular diseases (390-458, International Classification of Diseases, World Health Organization/Health Statistics/Eight Revision) at the levels of primary medical care, consultant medical care, community hospital care and teaching hospital care for a hypothetical region with a population of two million people that is increasing at an annual rate of 1.2 per cent.

The "Population not utilizing medical or hospital care" in this example includes that fraction of the total population not under medical or hospital care associated with cardiovascular diseases. It includes people with untreated cardiovascular diseases as well as people without these conditions. The primary medical care, consultant medical care, community hospital care and teaching hospital care states include people receiving each of these levels of care because they have a diagnosed cardiovascular disease.

PREDICTION

The following two tables illustrate the inputs for the prediction model.

INSERT TABLE 2.

Table 2 presents the empirical transitional probabilities, representing all possible flows among the health services states in this numerical example. Empirical transitional probabilities are the transitional probabilities calculated during the empirical time period, i.e. the unit of time over which the number people transiting from state i to state j has been calculated. If the empirical time periods were the same for all transitional probabilities, then the sum of those in the same row would add up to one.

In this table the referrals from primary and consultant medical care are the transitional probabilities for three month periods and those from community and teaching hospital care are daily transitional probabilities. The data on flow from the "Population not utilizing medical or hospital care" are annual transtitional probabilities. The use of different empirical time periods reflect the difficulty of obtaining adequate data but does not impair the logic supporting the model.

INSERT TABLE 3.

Table 3 illustrates the empirical estimates of the initial fractions of the total population in each of the health services states. The empirical estimates presented in this numerical example have been adapted from different sources[30,31,32,33]. The data is merely illustrative and no significance should be attached to the particular numbers used.

The outputs of the model are those fractions of the population in the different health services states that are calculated to be present at different time periods. If these fractions are known, the required manpower and facility resources for the total population can be calculated.

INSERT FIGURE IV.

The unbroken line in Figure IV shows the predicted number of physicians in primary and consultant medical care required for the exclusive care of cardiovascular conditions in the above mentioned population.

INSERT FIGURE V.

The unbroken line in Figure V shows the predicted number of community and teaching hospital beds required for the exclusive care of cardiovascular conditions in the same population.

SIMULATION

Simulation consists in studying the repercussions that changes in the system, associated with changes in the transitional probability matrix, have on utilization of health services, and the consequent requirements in resources at different time periods.

For example, suppose that, as a result of a proposed mass screening program for cardiovascular diseases, the number of persons with these diseases entering the primary medical care state during one year would double and the number referred for consultant medical care would increase by a quarter. Since the health services system is regarded as an interdependent whole rather than as the sum of its independent states, an administrator responsible for the health of the population in this region might ask for an estimate of the repercussions this change would have on the utilization of the various health services states and on their resource requirements.

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The situation is simulated in the Markovian model by multiplying the transitional probability of a patient with cardiovascular disease going from state "Population not utilizing medical or hospital care" to state "Primary medical care" by two and the transitional probability from the state "Population not utilizing medical and hospital care" to "Consultant medical care" by five fourths.

The output of the simulation model in this application is the estimated utilization indicated by the new fractions of the population in each health services state at different time periods. This new set of fractions will determine the new set of requirements.

The broken lines in Figure IV reflect the new manpower required as a result of the simulated situation.

The broken lines in Figure V reflect the new facility requirements in the simulated situation.

The simulated mass screening for cardiovascular diseases mentioned above in the population of two million people would require, for instance, at the end of five years the following additional manpower and facility resources for the exclusive care of patients with those conditions: 68 more primary care physicians, eight more specialists or consultants, 976 more community hospital beds and 172 more teaching hospital beds.

GOAL-SEEKING

Goal-seeking seeks to determine that alternative which will minimize a given constraint in order to reach a specified goal.

In the health services system described, planning services for the care of patients with cardiovascular diseases in the hypothetical region with two million people might include the goal of doubling in ten years the number of patients with cardiovascular diseases under primary medical care, while utilization of other levels of care remained the same. It might also include the knowledge that there will be few additional resources available at that time. The health services administrator might ask how current resources should be utilized, at different time periods, to reach the desired objective in such a way that minimal additional resources are required. In other words, the problem is to choose the utilization strategy that will minimize change in current resources to reach the specified goal.

In goal-seeking the input to the model is current utilization, defined by current fractions of the population in each state, desired utilization during the time designated, and the utilization parameters. A further input in the goal-seeking model is the constraint that must be minimized by the alternative chosen.

The output of the model would be the optimum utilization strategy, at different time periods, that will reach the specified goal with minimum change in current resources.

The dotted line in Figure IV shows the calculated number of primary and consultant medical care physicians required, at different time periods, in order to meet the goal presented in the goal-seeking model.

In Figure V, the dotted line shows the number of community and teaching hospital beds required, at different time periods, to meet the given goal.

OTHER APPLICATIONS

This Markovian model can be expanded to include two new states, birth and death, and different transitional probabilities matrices for each age group and thus, consider the different utilization rates of personal health services by different age groups. With this expansion, the model takes into account: first, changes in size and age structure of the population and second, the different utilization experiences of the different age groups[34].

SUMMARY

This chapter presents some of the mathematical analytical models used either in operations research or systems analysis that have been used in health services planning. Emphasis has been placed in detailing Markov models. A Markovian planning model is described as a tool for predicting requirements for resources, for calculating changes in those

resources in simulated situations, and for estimating the optimum alternative for the constraint chosen to reach a specified goal. A practical example describing the planning of personal health services for patients with cardiovascular diseases illustrates the three applications of the model.

APPENDIX 1

Estimating the Probability of Going from Health Services State i to State j During the Fundamental Time Interval

P_{ij}, the transitional probability, denotes the probability that a person in state i at the beginning of the empirical time period will go to state j during that period. The empirical time period, T_{ij}, is the unit of time over which the number of people, n_{ij} (T_{ij}), have moved from state i to state j.

$$[1] \qquad P_{ij} = \frac{n_{ij}\ (T_{ij})}{n_i}$$

The transitional probability, P_{ij}, given for different empirical time period, T_{ij}, are translated into daily transitional probabilities i.e. the time period of one day is selected as the fundamental time period because this is the period of time during which it is highly unlikely that a person will be in more than one state. Thus, the duplication in calculating the number of persons in each of the different states is avoided. The fundamental time period is the time period chosen to define the transitional probability matrix, input of the model. In this matrix, the sum of all elements in the same row add up to one.

The daily transitional probabilities are obtained from

$$[2] \qquad q_{ij} = \frac{P_{ij}\ (T_{ij})}{T_{ij}}, \qquad i \neq j$$

where,

q_{ij}	is the probability of going from state i to state j during a fundamental time interval of one day.
$P_{ij}(T_{ij})$	is the probability of going from state i to state j during the empirical time period T_{ij}, and,
T_{ij}	is the empirical time period in days.

The daily transitional probability matrix, Q, is defined by

$$Q = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1I} \\ q_{21} & q_{22} & \cdots & q_{2I} \\ & \cdots & & \\ q_{I1} & q_{I2} & \cdots & q_{II} \end{pmatrix}$$

APPENDIX 2

Calculating Resources Requirements

Knowing the health services state fractions $P_i(t)$, the manpower (MD = physicians) and facilities (BEDS) requirements are calculated from formulas [3] and [4].

[3] $$R_{MD_i}(t) = \frac{\frac{P_i(t).N(t)}{L_i} \times \gamma_i \times 365}{\Theta_i}$$

where,

$R_{MD_i}(t)$ is the number of physicians for state i (as a function of time t),

$P_i(t)$ is the fraction in state i at time t,

$N(t)$ is the size of population base, at time t, and determined by the rate of population growth,

L_i is the average length of stay in health services state i and is equal to the fraction $q_{ii}/(1-q_{ii})$,(q_{ii}, daily probability of remaining in state i),

γ_i is the number of visits per entry at state i.

$\frac{P_i(t).N(t)}{L_i}$ is the number of entries to state i per day.

It may help to clarify this formula if $P_i(t).N(t)$ is considered to be prevalence. Prevalence is then, equal to number of entries, i.e. incidence, per day multiplied by the average length of stay.

Altogether the numerator $\frac{P_i(t).N(t)}{L_i}$ x γ_i x 365 is the number of visits required for state i per year. θ_i, the denominator, is the average physician load factor or the number of visits at state i per physician per year.

Similarly, the requirements for beds is calculated with formula [4].

[4] $$R_{BEDS}(t) = \frac{P_i(t).N(t)}{F_i}$$

where,

F_i is the occupancy desired at state i.

APPENDIX 3

Calculating $P_i(t)$: Fractions of the Total Population in Health Services State i, at Time Period t.

The predicted fractions of the population (or probabilities of being in the states) in time period t is given by the expression [5].

[5] $$\overrightarrow{P(t)} \qquad \overrightarrow{P(0)}\, Q^t$$

where,

$\overrightarrow{P(t)}$	is the vector representing the fractions of the population in the different states, at time t days.
$\overrightarrow{P(0)}$	is the vector representing the initial fractions of the population in the different states.
Q	is the daily transitional probability matrix and
t	is the time in days from the initial period to the end of the t time period.

Thus, given P_i (0) and P_{ij}, one may predict P_i (t) using the Markovian assumptions.

In Simulation, the same mathematical model is used.

In Goal-Seeking, the problem solved is to minimize "the amount of change" subject to reaching the desired goal. This minimizing change is embodied in the selection of the objective function in a mathematical quadratic program.* The problem solved is:

*The quadratic program used in Goal Seeking has been programmed for the computer by Judith Liebman under PHS Grant HM (00279).

$$[6] \qquad \underset{\{P^1_{ij}\}}{\text{minimize}} \sum^{I} \sum^{I} W_{ij}(P^1_{ij} - P_{ij})^2$$

$$\text{subject to } \overrightarrow{P(\infty)} = \overrightarrow{P(\infty)} \cdot Q^1$$

The objective function is the weighted Euclidian distance between the solution "referral rates" P^1_{ij} and the current referral rates, P_{ij}. The problem then is to minimize the change of referral pattern necessary to effect a desired steady state vector $\overrightarrow{P(\infty)}$ of fractions of the population in the various health services states[35].

The terms "constraint" and "limitation" have been used in the text in some instances in place of "objective function", for purposes of comprehensibility to the non-mathematical reader. It is hoped that this flexibility of notation will not cause any confusion.

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28. The choice of states in this example is arbitrary. Another choice of states for instance, could have been "able", "disabled", and "dead", like in Sverdrup, E. "Estimates and Test Procedures in connection with Stochastic Models of Deaths, Recoveries, and Transfers between Different States of Health". Skand. Aktuar., 46:184, 1965, mentioned in reference 15, p. 73.

29. The term Simulation used in this application is different from the similar term used in Operations Research, which refers to a statistical sampling method. The application simulation, as defined in this paper, is a parametric study, in which, by varying the relevant transitional probabilities parametrically one may simulate the effect of changing the patterns of referral among two or more states. The method to calculate the fractions of the population at different time periods in the same as that used in prediction (Appendix 3).

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TABLE 1

TRANSITIONAL PROBABILITIES

EACH TRANSITIONAL PROBABILITY REPRESENTS A

FLOW BETWEEN TWO DIFFERENT HEALTH SERVICES STATES

j STATE / i STATE		STATE OF MEDICAL AND HOSPITAL CARE				
		POPULATION NOT UTILIZING MEDICAL OR HOSPITAL CARE	PRIMARY MEDICAL CARE	CONSULTANT MEDICAL CARE	COMMUNITY HOSPITAL CARE	TEACHING HOSPITAL CARE
STATE OF MEDICAL AND HOSPITAL CARE		1	2	3	4	5
POPULATION NOT UTILIZING MEDICAL OR HOSPITAL CARE	1	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}
PRIMARY MEDICAL CARE	2	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}
CONSULTANT MEDICAL CARE	3	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}
COMMUNITY HOSPITAL CARE	4	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}
TEACHING HOSPITAL CARE	5	P_{51}	P_{52}	P_{53}	P_{54}	P_{55}

TABLE 2

INPUTS IN THE PREDICTION MODEL

EMPIRICAL ESTIMATES OF THE TRANSITIONAL PROBABILITIES

j STATE / i STATE — STATE OF MEDICAL AND HOSPITAL CARE		STATE OF MEDICAL AND HOSPITAL CARE: POPULATION NOT UTILIZING MEDICAL OR HOSPITAL CARE	PRIMARY MEDICAL CARE	CONSULTANT MEDICAL CARE	COMMUNITY HOSPITAL CARE	TEACHING HOSPITAL CARE
		1	2	3	4	5
POPULATION NOT UTILIZING MEDICAL OR HOSPITAL CARE	1	.923	.035	.004	0	0
PRIMARY MEDICAL CARE	2	.324	.552	.076	.044	.008
CONSULTANT MEDICAL CARE	3	.528	.242	.182	.008	.032
COMMUNITY HOSPITAL CARE	4	.073	.054	.054	.800	.016
TEACHING HOSPITAL CARE	5	.115	.077	.098	.044	.662

No sumar / ver texto

TABLE 3

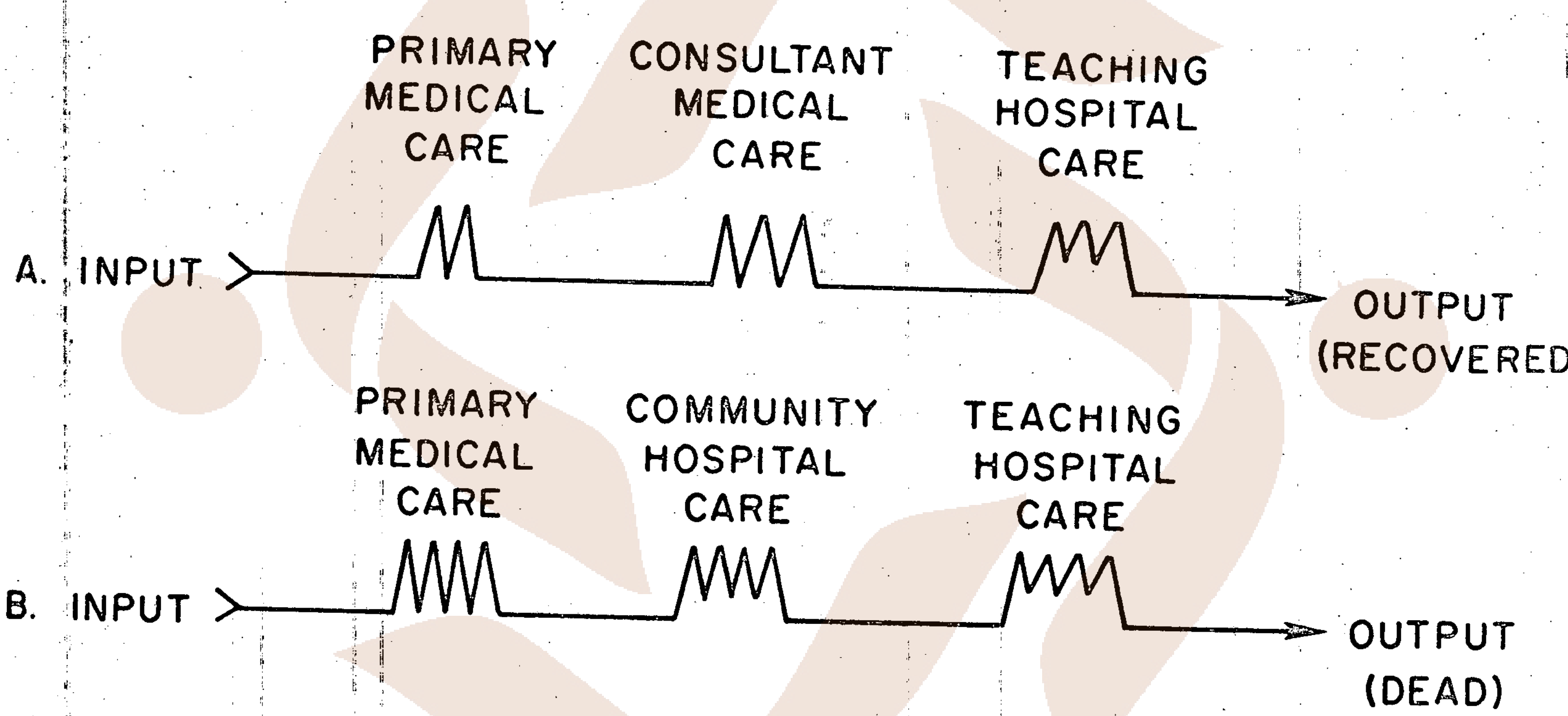
INPUTS IN THE PREDICTION MODEL

INITIAL FRACTIONS $P_i(0)$, AT STATE i, FOR CARDIOVASCULAR DISEASES.

STATE OF MEDICAL AND HOSPITAL CARE (i)	POPULATION NOT UTILIZING MEDICAL OR HOSPITAL CARE	PRIMARY MEDICAL CARE	CONSULTANT MEDICAL CARE	COMMUNITY HOSPITAL CARE	TEACHING HOSPITAL CARE
	1	2	3	4	5
$P_i(0)$	.9621	.0273	.0037	.0043	.0008

Figure I.

UTILIZATION STRATEGIES



A. 2 visits per entry to Primary Medical Care
3 visits per entry to Consultant Medical Care
3 days per entry to Teaching Hospital Care

B. 4 visits per entry to Primary Medical Care
4 days per entry to Community Hospital Care
4 days per entry to Teaching Hospital Care

Figure II.

STATES AND FLOWS

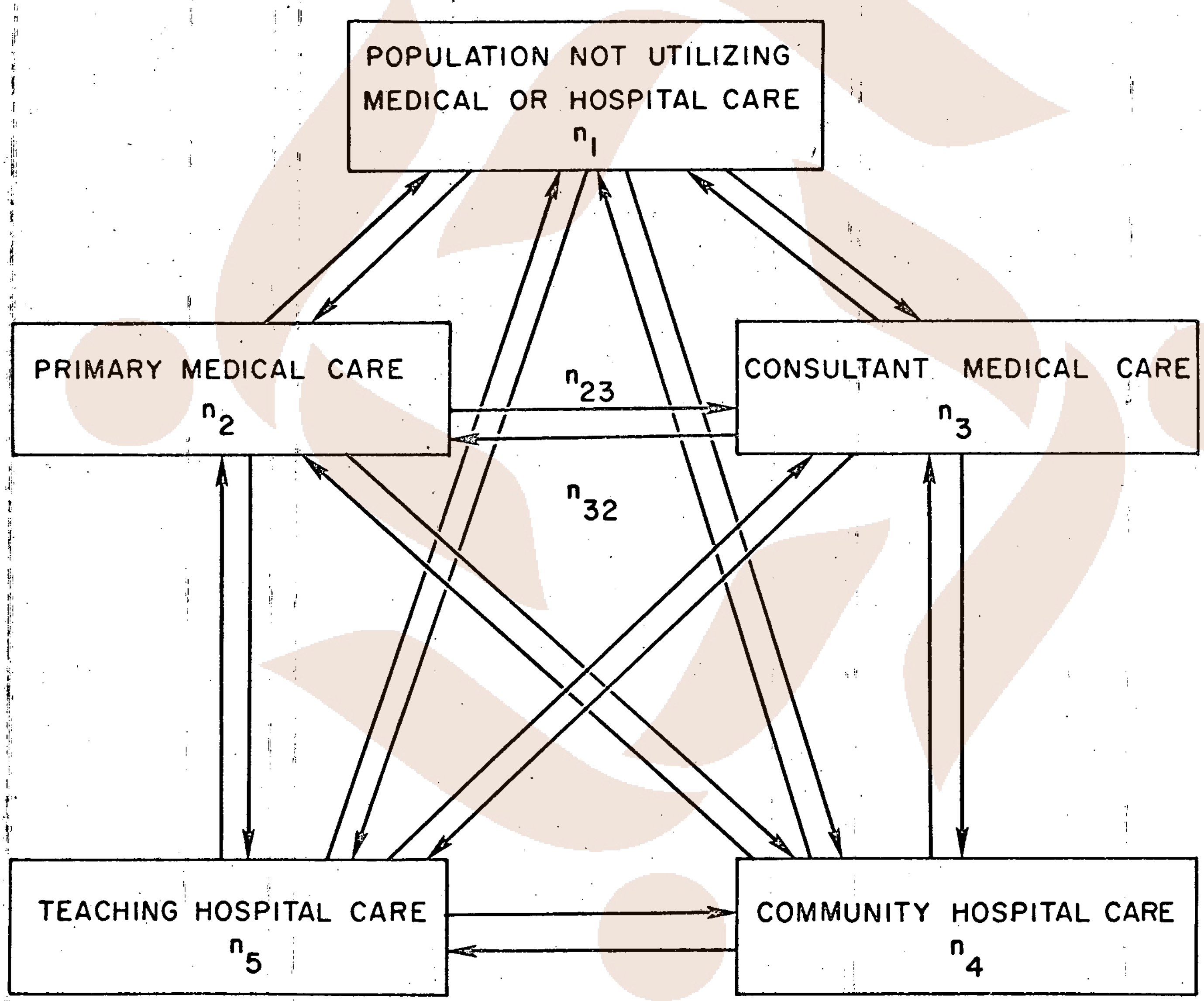


Figure III.

GRAPHICAL REPRESENTATION OF PREDICTION, SIMULATION AND GOAL SEEKING

The encircled elements are the outputs, the uncircled are the inputs.

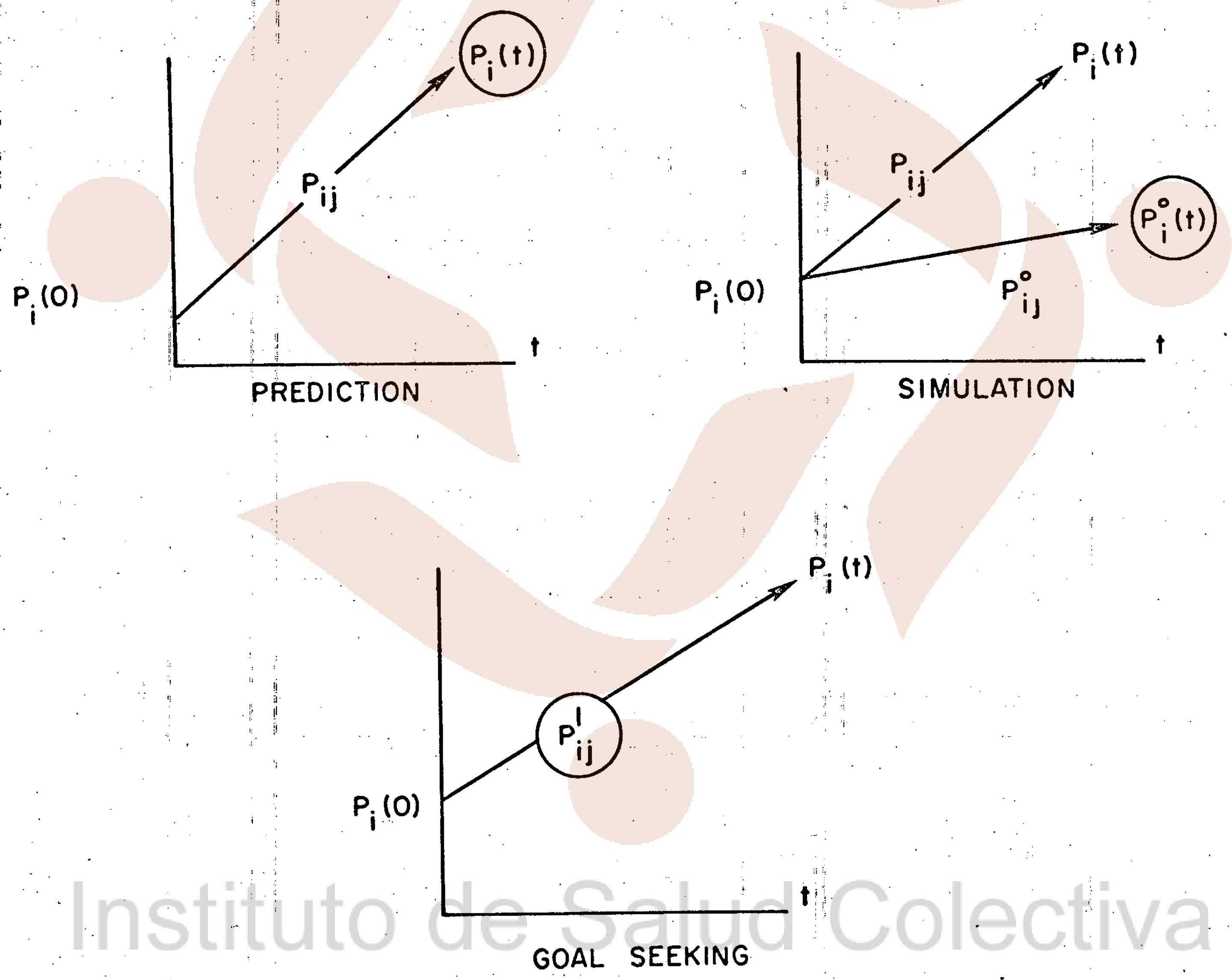


Figure IV.

MANPOWER REQUIREMENTS IN PRIMARY AND CONSULTANT MEDICAL CARE

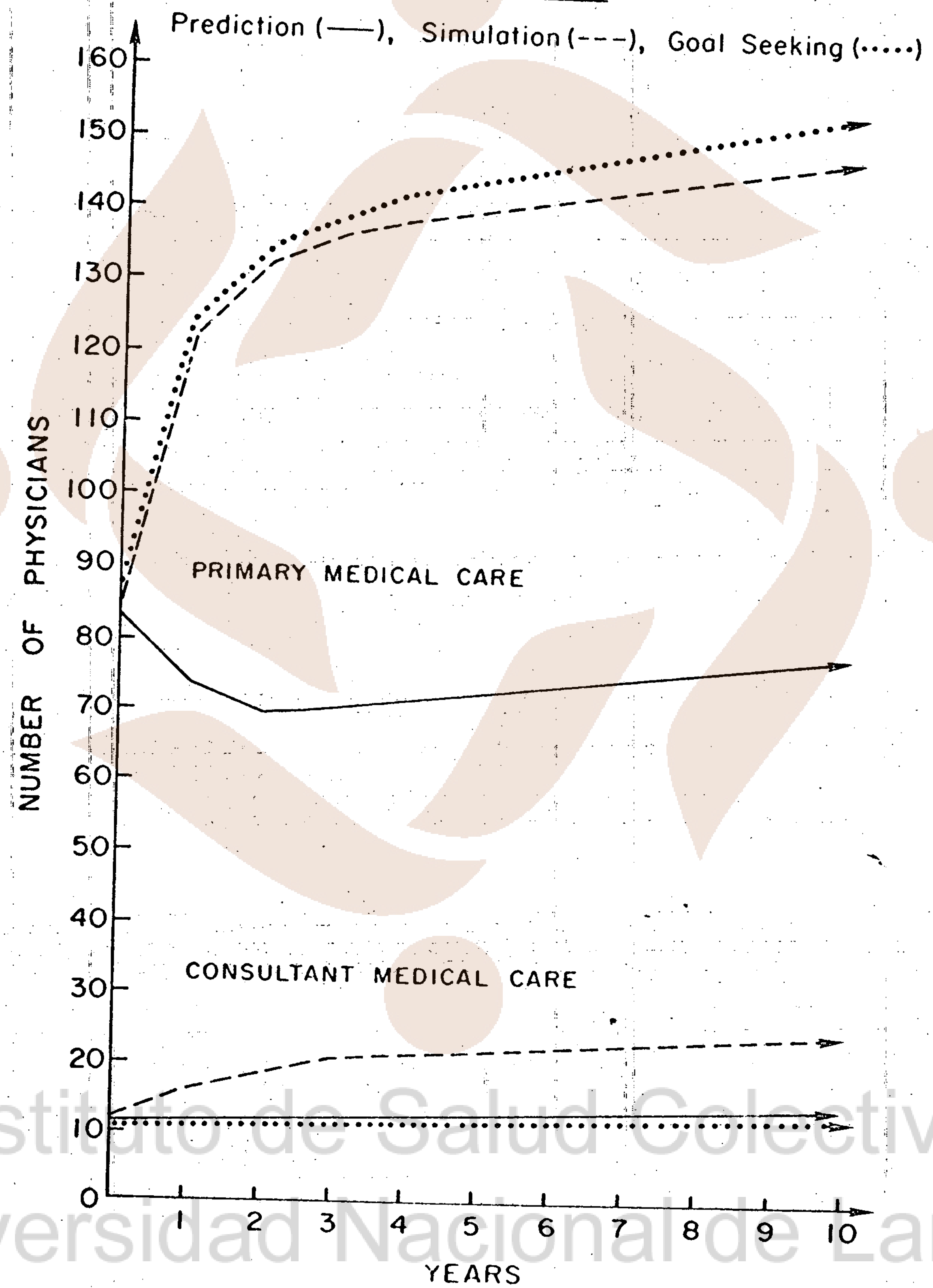


Figure V.

HOSPITAL BEDS REQUIREMENT IN COMMUNITY AND TEACHING HOSPITAL CARE

Prediction (——), Simulation (---), Goal Seeking (·····)