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A MATHEMATICAL MODEL FOR HEALTH PLANNING: PREDICTION, SIMULATION, AND GOAL SEEKING

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SUMMARY

A Markovian planning model is described as a tool for predicting requirements for resources, for calculating changes in those resources in simulated situations, and for estimating the optimum alternative for the constraint chosen to reach a specified goal. A practical example, describing the planning of personal health services for patients with cardiovascular diseases, illustrates the three applications of the model.

A system is defined as a "regularly interacting or interdependent group of items forming a unified whole ... as ... : a group of interacting bodies under the influence of related forces ..." (Webster's Seventh New Collegiate Dictionary (1967).) A personal health services system consists of interdependent elements, such as physicians, nurses, facilities and other resources, that interact under the influence of diverse forces with the community they serve.

The elements of any health services system can be grouped in subsystems that depend on the criteria for the grouping, e.g., first contact care or primary medical care, specialist care or consultant medical care, community hospital care, teaching hospital care, etc. The term subsystem is interchangeable with the term level or state of care.

The state of a health services system can be defined by the values of those variables that describe its elements, e.g., prevalence of a particular disease, available hospital beds, etc., as well as by the process of transformation in the system whereby inputs are translated into outputs.

The **input** into each state can be measured by the number of entries, i.e. persons or conditions, as determined by the actual "demand" or "need" for services. The parameters that define these inputs will depend on the criteria used to define such entities as disease, disability, and discomfort. (White, K. L. "Improved Medical Care Statistics and the Health Services System." (1967).)

The output of each state can be measured by the number of discharges or outcomes. Alternative outcomes might include dead or alive, disesaed or recovered, disabled or fit and uncomfortable or comfortable.

The **throughput** represents the movement of patients through the several states of the system. This movement within the system can be within the same subsystem, i.e., a transfer, or between two different subsystems, i.e. a referral.

Transfers and referrals document the movement or flow of people within a health services system and the dynamic relationships among its different states and units. The series of referrals and transfers experienced by each patient defines his utilisation experience and reflects the **utilisation strategy.** (Reinke, W. A. Quantitative Decision Procedures. (in press)

The throughput of the entire system can be defined as the totality of utilisation strategies for all patients. (Fig. I).

The method of planning presented below requires knowledge of the internal relationships among a system's parts and therefore requires information about the static aspects of the system, i.e., enumeration of the system's parts and the measurement of their activity, e.g., the number of services provided, and about its dynamic aspects, as reflected by information about the referrals and transfers.

DESCRIPTION OF THE MODEL

This model is based upon the Markovian process, (Loéve, M. M., Probability Theory. (1963); Bartholomew, D. J., Stochastic Models for Social Process. (1967); Kemeny, T. G. **et al.**, Finite Markov Chains. (1960.), commonly used in system analysis. This is a process in which the health services states are postulated and the probabilities of going from one state to another, called the transitional probabilities, determine the number of people in various states over a period of time. (Navarro, V., "Planning Personal Health Services: A Markovian Model." (1968); Navarro, V. **et al.**, "A Planning Model for Personal Health Services" (1968); Hope, K. **et al.**, "Patients Flows in the National Health Services: A Markov Analysis." (1968).).

In the present application the assumption is made that every person in the population of a defined geographical region is characterised as belonging to one, and only one, of several mutually exclusive states of a health service system at any point in time. (Fig. II).

The health services states shown in Fig. II have been chosen arbitrarily. The state described as "Population not utilising medical or hospital care" includes all persons who are not in any other state. It includes healthy persons as well as sick persons who are not under medical care in any of the other states. Primary medical care, consultant medical care, community hospital care and teaching hospital care states contain people receiving these levels of care respectively. The number of states could be extended by adding other states of care as well as different units within each state. The size of the model can be extended in accord with the complexity and comprehensiveness desired and the availability of usable information. The population to be examined can be defined by demographic and/or epidemiological criteria.

In Fig. II to say that n_2 equals 20 persons means that at this moment, $t = 0$, there are 20 persons under primary medical care.

The fractions of the population at each state, at different time periods, $P_i(t)$, equals the number of people in that health services state at that time, $n_i(t)$, divided by the total number of people in the region served by the system at that time, $N(t)$.

For a stationary Markovian chain analysis, the transitional probabilities of going from one state to another during the fundamental time period must be calculated. The transitional probability, P_{ij}, equals the number of people, n_{ij}, who are transferred from state i to state j during the defined time period, divided by the number of people, n_i, in state i at the beginning of that period. For example, if n_4, the number of people under community hospital care at a beginning point in time, is 200, and n_{45}, the number of referrals from community hospital care to teaching hospital care measured in the week that follows, is 50, then $P_{45} = 50/200 = 0.25$ is the measured transitional probability per week of going from community to teaching hospital.

This transitional probability denotes the probability that a person who is in state i at the beginning of the defined time period will go to state j during that period. Thus it defines the movement of people within the system and reflects functional relationship among the states (Table 1).

Table 1 presents the transitional probabilites in the described model. Each transitional probability represents a flow between two different health services states. P_{24}, for instance, represents the probability that a person is in the primary medical care state at the beginning of the chosen time period and goes to the state of community hospital care during the period.

In this model P_{ij} is taken as known. It is determined from information about referrals within the system. (Appendix 1).

If the fractions of the population in different health services states are known and if the parameters that define productivity are known, the manpower and facilities required can be calculated. (Appendix 2).

APPLICATIONS OF THE MODEL
PREDICTION

Prediction is the ordinary statistical problem of forecasting. At the simplest level it involves extrapolation of past experiences into the future.

In the Markovian model, when the transitional probabilities are known, prediction is possible when only the initial fraction of the population in each state is known. (Appendix 3). Prediction involves calculating the fractions of the population expected to be in the several health services states at different time periods in the future. The inputs for this model of prediction are the known current fractions in each state and the transitional probability matrix that reflects the dynamics of the system. The outputs of the model are the estimated fractions of the population in each state at different time periods in the future. If the productivity of current resources is known it is possible to estimate the manpower and facilities required in each state in the future.

SIMULATION

Simulation involves observation of changes in the health services system and the repercussions of those changes on present and future utilisation and resources. The inputs of the model applied to simulation are the fractions of the population currently in each state and the new set of transitional probabilities that reflects simulated changes in the system. The outputs are the new patterns of utilisation determined by the changes. Since the productivity of the resources is known, these new fractions in each state can be translated into a new set of resources.

GOAL-SEEKING

Goal-seeking involves determining that alternative which minimises "costs" or "changes" in resources, required to achieve, in a given time period, specified utilisation patterns or specified needs for resources.

The inputs of the Markovian model in goal-seeking are: the present fractions of the population in each state, the desired future steady state fraction in each state (or the desired number of resources in a particular state), and the selected constraint, e.g., a cost constraint, a resource constraint, etc., that the alternative specified must meet. The problem is to choose that alternative, defined by a transitional probability matrix, which will minimise the constraint selected. Actually there will be an infinite number of possible alternatives in going from the present level of utilisation to one desired in the future, but only one alternative will minimise the constraint selected. For instance, if the constraint is "cost", then the alternative chosen will be the one that minimises the cost of going from the present to the desired future level of utilisation. Another example of constraint might be "minimising changes", and, in that case, the alternative chosen would be the one which would require minimal additional resources for each health service state at different future time periods. (Fig. III) Fig. III illustrates the three applications of the Markovian model.

THE MARKOVIAN MODEL APPLIED TO PLANNING FOR CARDIOVASCULAR DISEASES

This example deals with the application of the Markovian model described to the planning of personal health services for patients with cardiovascular diseases (390—458, International Classification of Diseases, World Health Organisation/Health Statistics/ Eighth Revision) at the levels of primary medical care, consultant medical care, community hospital care and teaching hospital care for a hypothetical region with a population of two million people that is increasing at an annual rate of 1.2 per cent.

The "Population not utilising medical or hospital care" in this example includes that fraction of the total population not under medical or hospital care associated with cardiovascular diseases. It includes people with untreated cardiovascular diseases as well as people without these conditions. The primary medical care, consultant medical care, community hospital care and teaching hospital care states include people receiving each of these levels of care because they have a diagnosed cardiovascular disease.

PREDICTION

The following two tables illustrate the inputs for the prediction model. (Table 2) Table 2 presents the transitional probability matrix, representing all possible flows among the health services states in this numerical example.

In this matrix the referrals from primary and consultant medical care are the transitional probabilities for three month periods and those from community and teaching hospital care are daily transitional probabilities.

The data on flow from the "Population not utilising medical or hospital care" are annual transitional probabilities. The use of different time periods reflects the difficulty of obtaining adequate data but does not impair the logic supporting the model. (Table 3)

Table 3 illustrates the empirical estimates of the initial fractions of the total population in each of the health services states. The empirical estimates presented in this numerical example have been adapted from different sources. (White, K. L. **et al**, "The Distribution of Cardiovascular Disease in the Community" (1963); Scottish Hospital In-Patient Statistics 1965, Scottish Home and Health Department, 1967; Ministry of Health and General Register Office, Report on Hospital In-Patient Enquiry for the Year 1960; Densen, P. M. **et al**, "Medical Care Plans as a Source of Morbidity Data" (1960).). The data are merely illustrative and no significance should be attached to the particular numbers used.

The outputs of the model are the fractions of people that are calculated to be in the different health services states at different time periods. If these fractions are known, the required manpower and facility resources for the total population can be calculated. (Fig. IV)

The unbroken line in Fig. IV shows the predicted number of physicians in primary and consultant medical care required for the exclusive care of cardiovascular conditions in the above-mentioned population. (Fig. V)

The unbroken line in Fig. V shows the predicted number of community and teaching hospital beds required for the exclusive care of cardiovascular conditions in the same population.

SIMULATION

Simulation consists in studying the repercussions that changes in the system, associated with changes in the transitional probability matrix, have on utilisation of health services, and the consequent requirements in resources at different time periods.

For example, suppose that as a result of a proposed mass screening program for cardiovascular diseases, the number of persons with these diseases entering the primary medicial care state during one year would double and the number referred for consultant medical care would increase by a quarter. Since the health services system is regarded as an inter-dependent whole rather than as the sum of its independent states, an administrator responsible for the health of the population in this region might ask for an estimate of the repercussions this change would have on the utilisation of the various health services states and on their resource requirements.

The situation is simulated in the Markovian model by multiplying the transitional probability of a patient with cardiovascular disease going from state "Population not utilising medical or hospital care" to state „Primary medical care" by two and the transitional probability from the state "Population not utilising medical and hospital care" to "Consultant medical care" by five-fourths.

The output of the simulation model in this application is the estimated utilisation indicated by the new fractions of the population in each health services state at different time periods. This new set of fractions will determine the new set of requirements.

The broken lines in Fig. IV reflect the new manpower required as a result of the simulated situation.

The broken lines in Fig. V reflect the new facility requirements in the simulated situation.

The simulated mass screening for cardiovascular diseases mentioned above in the population of two million people would require, for instance, at the end of five years the following additional manpower and facility resources for the exclusive care of patients with those conditions: **68 more primary care physicians, eight more specialists or consultants, 976 more community hospital beds and 172 more teaching hospital beds.**

GOAL-SEEKING

Goal-seeking seeks to determine the alternative which will minimise a given constraint in order to reach a specified goal.

In the health services system described, planning services for the care of patients with cardiovascular diseases in the hypothetical region with two million people might include the goal of doubling in ten years the number of patients with cardiovascular diseases under primary care, while utilisation of other levels of care remained the same. It might also include the knowledge that there will be a few additional resources available at that time. The responsible administrator might ask how current resources should be utilised at different time periods, to reach the desired objective in such a way that minimal additional resources are required. In other words, the problem is to choose the utilisation strategy that will minimise change in current resources to reach the specified goal.

In goal-seeking, the input to the model is current utilisation, defined by current fractions of the population in each state, desired utilisation during the time designated, and the utilisation parameters. A further input in the goal-seeking model is the constraint that must be minimised by the alternative chosen.

The output of the model would be the optimum utilisation strategy, at different time periods, that will reach the specified goal with minimum change in current resources.

The dotted line in Fig. IV shows the calculated number of primary and consultant medical care physicians required, at different time periods, in order to meet the goal presented in the goal-seeking model.

In Fig. V, the dotted line shows the calculated number of community and teaching hospital beds required, at different time periods, to meet the given goal.

ACKNOWLEDGMENTS

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APPENDIX 1

Estimating P_{ij} the Probability of Going from Health Services State i to State j During the Fundamental Time Interval

Let q_{ij} be the probability of going from state i to state j during a time interval of one day, and let $P_i [L = d]$ be the probability that the length of stay in health services state i is exactly d days.

Then,

$$P_i [L = d] = q^d_{ii} (1 - q_{ii})$$

and the average length of stay L_i is given by

$$L_i = \sum_{d=0}^{\infty} d \cdot P_i [L = d] = \frac{q_{ii}}{1 - q_{ii}} \qquad [1]$$

Let,

T_i be the fundamental time interval over which the transitional probabilities P_{ij}, $j = 1$, I should be estimated,

$a_{ij} (T_i)$ be the probability of going from state i to health state j during the time interval T_i, and $a_{ij}(T_i)$ are the empirical estimates which are input to the models, then,

[2] $$q_{ij} = \frac{a_{ij}(T_i)}{T_i},$$

for $i \neq j$, and

[3] $$q_{ii} = 1 - \sum_{j \neq i} q_{ij}.$$

Let matrix Q be defined by,

$$Q = \begin{pmatrix} q_{11} & q_{12} & \dots\dots & q_{1I} \\ q_{21} & q_{22} & \dots\dots & q_{2I} \\ \dots & \dots & \dots\dots & \dots \\ q_{I1} & q_{I2} & \dots\dots & q_{II} \end{pmatrix}$$

Matrix T is then given by,

[4] $$T = Q^{365}$$

where

$$T = \begin{pmatrix} P_{11} & P_{12} & \dots\dots & P_{1I} \\ P_{21} & P_{22} & \dots\dots & P_{2I} \\ \dots & \dots & \dots\dots & \dots \\ P_{I1} & P_{I2} & \dots\dots & P_{II} \end{pmatrix}$$

and P_{ij} is the probability of going from state i at the beginning to state j at the end of the fundamental time interval, which has here been taken as one year.

APPENDIX 2

Calculating Resources Requirements

Knowing the health services state fractions $P_i(t)$, the manpower (MD = physicians) and facilities (BEDS) requirements are calculated from formula [5] and [6].

[5] $$R_{MD_i}(t) = \frac{\dfrac{P_i(t) \cdot N(t)}{L_i} \times \gamma_i \times 365}{0_i}$$

where,

$R_{MD_i}(t)$ is the required number of physicians for state i (as a function of time t),

$P_i(t)$ is the fraction in state i at time t,

$N(t)$ is the size of population base, at time t, and determined by the rate of population growth,

L_i is the average length of stay in health services state i and is equal to the fraction $q_{ii}/(1 - q_{ii})$, (q_{ii}, daily probability of remaining in state i), and

γ_i is the number of visits per entry at state i.

$$\frac{P_i(t) \cdot N(t)}{L_i}$$ is the number of entries to state i per day.

It may help to clarify this formula if $P_i(t) \cdot N(t)$ is considered to be prevalence. Prevalence is then, equal to number of entries, i.e. incidence, per day multiplied by the average length of stay.

Altogether the numerator $\frac{P_i(t) \cdot N(t)}{L_i} \times \gamma_i \times 365$ is the number of visits required for state i per year. θ_i, the denominator, is the average physician load factor or the number of visits at state i per physician per year.

Similarly, the requirements for beds is calculated with formula [6].

$$R_{BEDS}(t) = \frac{P_i(t) \cdot N(t)}{F_i} \quad [6]$$

where,

F_i is the occupancy desired at state i.

APPENDIX 3

Calculating $P_i(t)$: Fractions of the Total Population in Health Services State i, at Time Period t.

In the Markovian model, if the transitional probabilities P_{ij} are known, then the prediction problem can be solved knowing only $P_i(O)$.

If $t = 1$ week, for instance, the fraction of the total population in state i at the end of the week would be equal to the number of people staying in that state during the week plus the new entries from the other states.

For,

$$P_i(1) = P_{1i} P_1(0) + P_{2i} P_2(0) + \ldots + P_{Ii} P_I(0) \quad [7]$$

where $i = 1, I$, denoting the total number of health service states as I; i. e., the probability of being in state i at time 1 is equal to the probability of going from state 1 to state i during the time interval multiplied by the probability of being in the first state at time 0, plus the probability of going from state 2 to state i during the time interval multiplied by the probability of being in the second state at time 0, etc.

If matrix notation, expression [7] is given by,

$$P(1) = P(0)\,T \quad [8]$$

where,

$P((1) = P_1(1), P_2(1), \ldots, P_I(1))$
$P((0) = P_1(0), P_2(0), \ldots, P_I(0))$ and

$$T = \begin{pmatrix} P_{11} & P_{12} & \ldots\ldots & P_{1I} \\ P_{21} & P_{22} & \ldots\ldots & P_{2I} \\ \ldots & \ldots & \ldots\ldots & \ldots \\ P_{I1} & P_{I2} & \ldots\ldots & P_{II} \end{pmatrix}$$

Similarly,

$$P(n) = P(0)\,T^n, \quad [9]$$

where,

$P(n) = (P_1(n), P_2(n), \ldots, P_I(n)$

Thus, given $P_i(0)$ and P_{ij} one may calculate $P_i(n)$ using the Markovian assumption.

In Simulation, a similar approach is used.

In Goal-seeking, a quadratic programming approach is used. The problem solved is:

$$\text{minimize} \quad \sum_{i=1}^{I} \sum_{j=1}^{I} W_{ij} (P_{ij} - P^{\circ}_{ij})^2,$$

subject to,

[10] $$P(\infty) = P(\infty)\,T$$

The constraint [10] assures the solution, referring pattern, T has the desired steady state of health services utilization $P(\infty)$. The minimization involves finding that referral pattern T which comes closest (in the sense of weighted distance) to a given actual referral pattern T°. The term "constraint" has been used in the text in some instances in place of "objective function," for purposes of comprehensibility to the non-mathematical reader. It is hoped that this flexibility of notation will not cause any confusion.

TABLE 1

Transitional probabilities

EACH TRANSITIONAL PROBABILITY REPRESENTS A FLOW BETWEEN TWO DIFFERENT HEALTH SERVICES STATES

j STATE / i STATE — State of medical and hospital care		State of medical and hospital care: Population not utilizing medical or hospital care	Primary medical care	Consultant medical care	Community hospital care	Teaching hospital care
		1	2	3	4	5
Population not utilizing medical or hospital care	1	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}
Primary medical care	2	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}
Consultant medical care	3	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}
Community hospital care	4	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}
Teaching hospital care	5	P_{51}	P_{52}	P_{53}	P_{54}	P_{55}

TABLE 2

Inputs in the prediction model

EMPIRICAL ESTIMATES OF THE TRANSITIONAL PROBABILITIES

j STATE / i STATE — State of medical and hospital care		State of medical and hospital care				
		Population not utilizing medical or hospital care	Primary medical care	Consultant medical care	Community hospital care	Teaching hospital care
		1	2	3	4	5
Population not utilizing medical or hospital care	1	.923	.035	.004	0	0
Primary medical care	2	.324	.552	.076	.044	.008
Consultant medical care	3	.528	.242	.182	.008	.032
Community hospital care	4	.073	.054	.054	.800	.016
Teaching hospital care	5	.115	.077	.098	.044	.662

TABLE 3

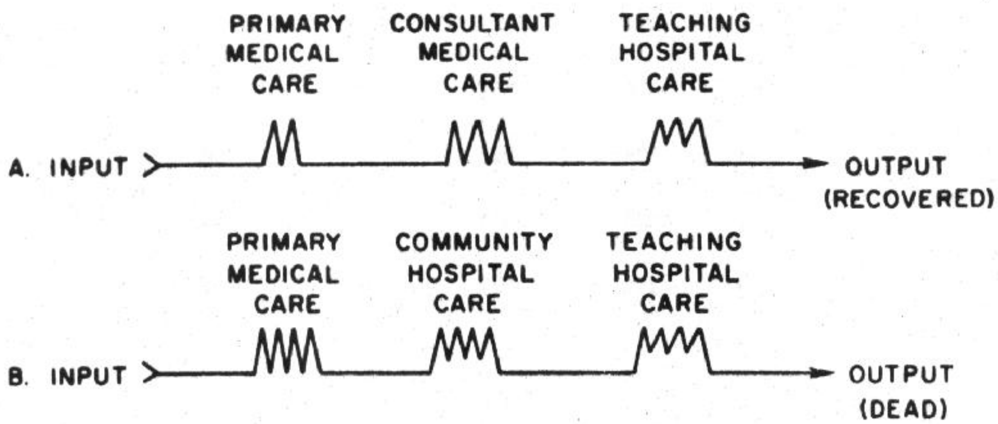
Inputs in the prediction model

INITIAL FRACTIONS $P_i(0)$, AT STATE i, FOR CARDIOVASCULAR DISEASES

State of medical and hospital care (i)	Population not utilizing medical or hospital care	Primary medical care	Consultant medical care	Community hospital care	Teaching hospital care
	1	2	3	4	5
$P_i(0)$	.9621	.0273	.0037	.0043	.0008

Figure I.

UTILIZATION STRATEGIES



A. 2 visits per entry to Primary Medical Care
3 visits per entry to Consultant Medical Care
3 days per entry to Teaching Hospital Care

B. 4 visits per entry to Primary Medical Care
4 days per entry to Community Hospital Care
4 days per entry to Teaching Hospital Care

Figure II.

STATES AND FLOWS

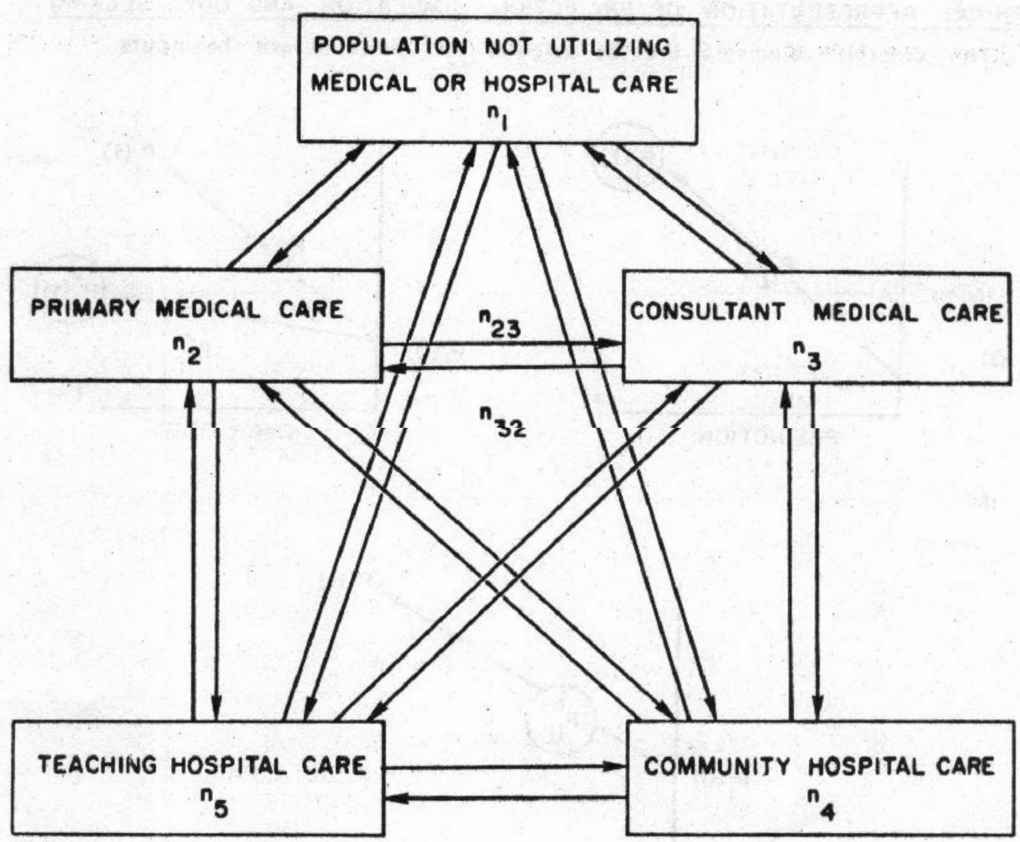


Figure III

GRAPHICAL REPRESENTATION OF PREDICTION, SIMULATION AND GOAL SEEKING

The encircled elements are the outputs, the uncircled are the inputs.

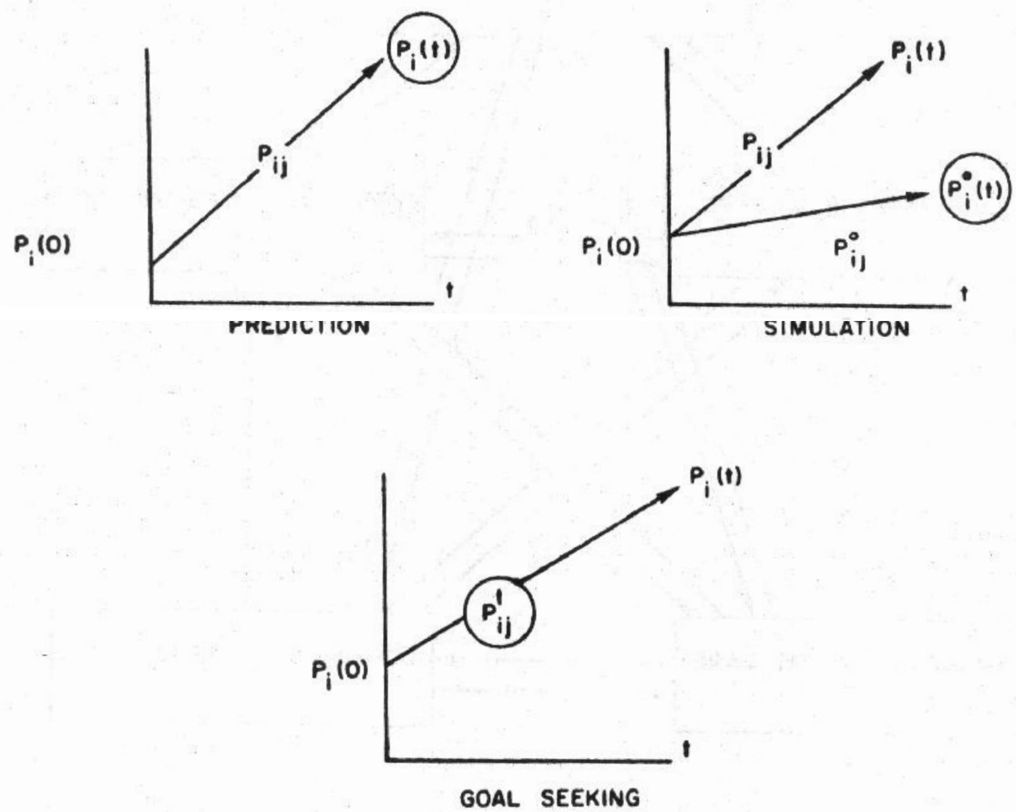


Figure IV.

MANPOWER REQUIREMENTS IN PRIMARY AND CONSULTANT MEDICAL CARE

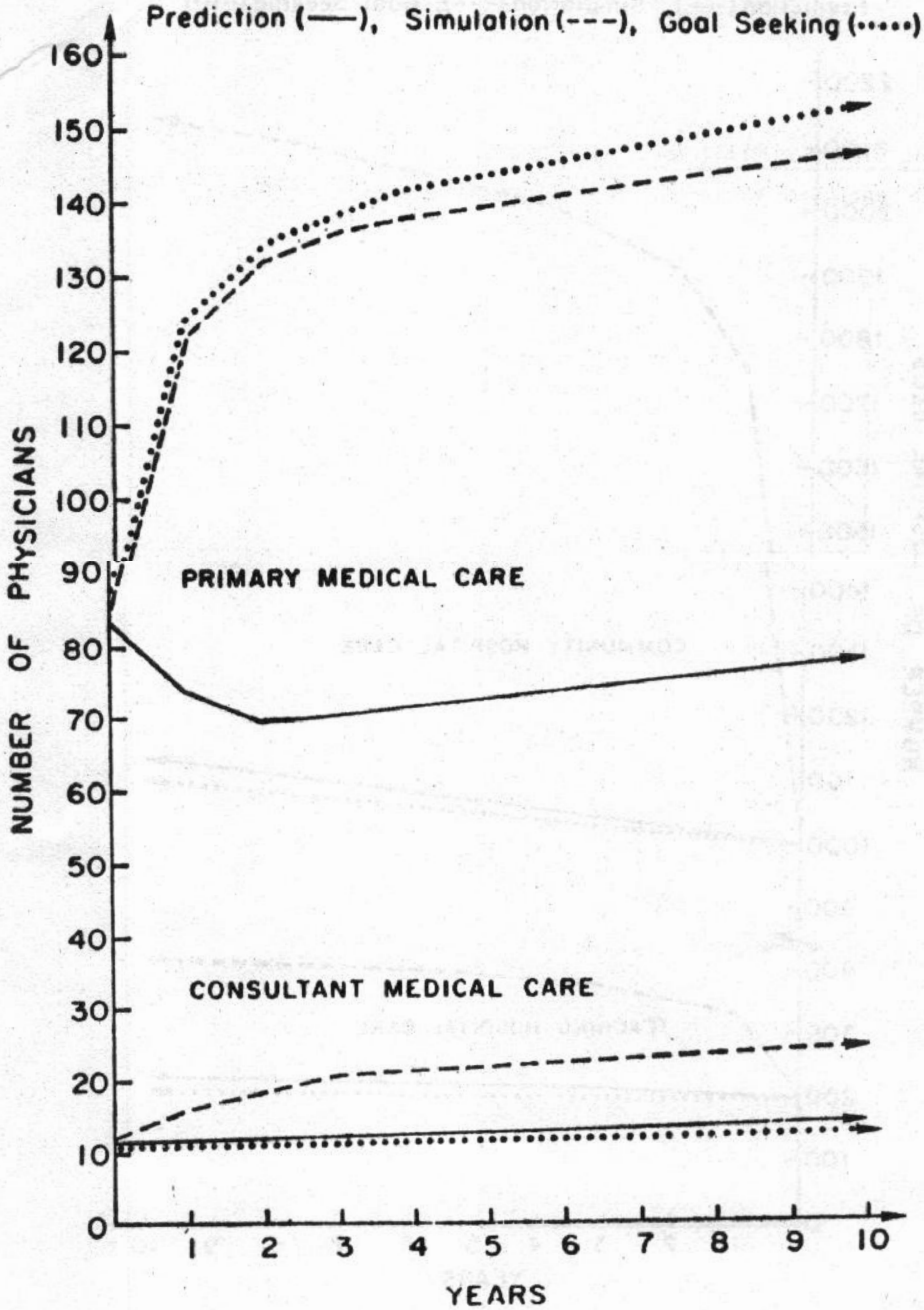


Figure V

HOSPITAL BEDS REQUIREMENT IN COMMUNITY AND TEACHING HOSPITAL CARE

Prediction (—), Simulation (---), Goal Seeking (·····)